The 44th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

Written Round: February 22, 2025 at Regional Testing Sites

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GEOMETRY EXAMINATION

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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions are not arranged in order of difficulty. For each question, choose the best of the five options labeled A, B, C, D and E. Calculators are NOT permitted.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a paper with: all questions answered correctly earns a score of 250, all questions left blank earns a score of 50, and all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.
- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Editing by Alejandro Ginory, University of Alabama in Huntsville
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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Analysis
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, Northrop Grumman, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. Two opposite sides of a square are increased by 25% and the other two sides are decreased by 60%. What is the percent decrease in the area of the resulting rectangle?

(**A**) 20% (**B**) 30% (**C**) 40% (**D**) 50% (**E**) None of these

2. Let T be the set of points in the xy-plane satisfying $|x| + |y| \leq 4$. What is the area of T?

(**A**) 8 (**B**) 12 (**C**) 16 (**D**) 32 (**E**) None of these

3. Find the perimeter of a regular polygon with sides of length $16u$ and interior angles measuring $162°$.

(**A**) $288u$ (**B**) $320u$ (**C**) $352u$ (**D**) $384u$ (**E**) None of these

4. What is the area of the circle defined by:

$$8x^2 + 8y^2 - 32x - 128y = 104$$

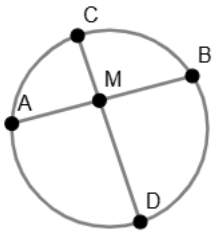
(**A**) 49π (**B**) 64π (**C**) 81π (**D**) 100π (**E**) None of these

5. The side of a square is the same length as the altitude of an equilateral triangle. Determine the ratio of the area of the square to the area of the equilateral triangle.

(**A**) $\sqrt{3}$ (**B**) $2\sqrt{3}$ (**C**) $3\sqrt{3}$ (**D**) $4\sqrt{3}$ (**E**) None of these

6. In the circle shown, $AB = 24$ and the perpendicular chord $\overline{CD}$ bisects $\overline{AB}$. Find the length of $\overline{BD}$ if $\overline{DM}$ is four times as long as $\overline{CM}$?

(**A**) $4\sqrt{5}$ (**B**) $8\sqrt{5}$ (**C**) $12\sqrt{5}$ (**D**) $16\sqrt{5}$ (**E**) None of these



7. The measure of the interior angles of a polygon are in an arithmetic progression with the least angle of $120°$ and a common difference of $5°$. What is the least number of sides that this polygon can have?

(**A**) 8 (**B**) 11 (**C**) 14 (**D**) 17 (**E**) None of these

8. The width, length, and height of a rectangular box measures 4 by 5 by 2 meters. Determine the length of the longest pole that can fit into the box.

(**A**) $\sqrt{5}$ m (**B**) $2\sqrt{5}$ m (**C**) $3\sqrt{5}$ m (**D**) $4\sqrt{5}$ m (**E**) None of these

9. Find the area of the rhombus $ABCD$ if $AB = 6$ and $m\angle DAB = 120°$.

(**A**) 9 (**B**) $9\sqrt{3}$ (**C**) 18 (**D**) $18\sqrt{3}$ (**E**) None of these

10. In triangle $\triangle ABC$, the lengths of the sides are $BC = 12$, $AC = 18$, and $AB = 20$. If D is a point on $\overline{AB}$ such that $\overrightarrow{CD}$ bisects $\angle C$, then find CD.

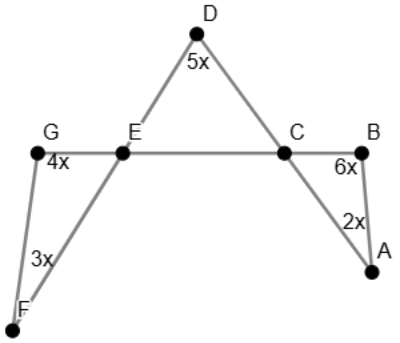
(**A**) $2\sqrt{30}$ (**B**) 13 (**C**) $4\sqrt{30}$ (**D**) 15 (**E**) None of these

11. Suppose a regular hexagon is inscribed in a circle of radius 2. Find the area of the hexagon.

(**A**) $\frac{2\sqrt{3}}{3}$ (**B**) $\frac{4\sqrt{3}}{3}$ (**C**) $6\sqrt{3}$ (**D**) $16\sqrt{3}$ (**E**) None of these

12. In the figure shown, what is the value of $m\angle ABC$? (Here x is measured in degrees in the figure.)

(**A**) 72° (**B**) 108° (**C**) 120° (**D**) 144° (**E**) None of these



13. If the radius of a sphere is doubled, the volume is made:

(**A**) 4 times as great (**B**) 6 times as great

(**C**) 8 times as great (**D**) 12 times as great (**E**) None of these

14. Find the sum of the lengths of the diagonals of a rectangle whose dimensions are 10 units by 24 units.

(**A**) 30 units (**B**) 36 units (**C**) 42 units (**D**) 48 units (**E**) None of these

15. What is the equation of the perpendicular bisector of the segment $\overline{AB}$ with endpoints $A(-12, 15)$ and $B(4, -3)$?

(**A**) $y = \frac{8}{9}x + \frac{86}{9}$ (**B**) $y = \frac{9}{8}x - \frac{86}{9}$ (**C**) $y = -\frac{8}{9}x - \frac{86}{9}$ (**D**) $y = -\frac{9}{8}x + \frac{86}{9}$ (**E**) None of these

16. What is the area of the triangle that has sides of length, 5, 10, and 13?

(**A**) $2\sqrt{14}$ (**B**) $6\sqrt{14}$ (**C**) $18\sqrt{14}$ (**D**) $24\sqrt{14}$ (**E**) None of these

17. How many faces does a polyhedral solid with 32 vertices and 60 edges have?

(**A**) 64 (**B**) 56 (**C**) 48 (**D**) 30 (**E**) None of these

18. A chord is perpendicular to a diameter of a circle at a point which divides the diameter into segments having the ratio 1 : 3. In what ratio does the chord divide the circumference?

(**A**) 2 : 3 (**B**) 3 : 4 (**C**) 1 : 2 (**D**) 1 : 4 (**E**) None of these

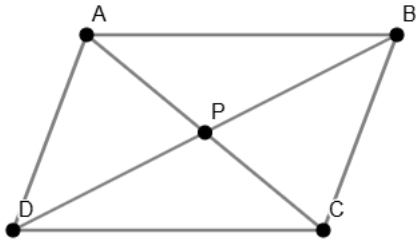
19. Determine the surface area of a sphere whose volume is 288π.

(**A**) 120π (**B**) 144π (**C**) 168π (**D**) 192π (**E**) None of these

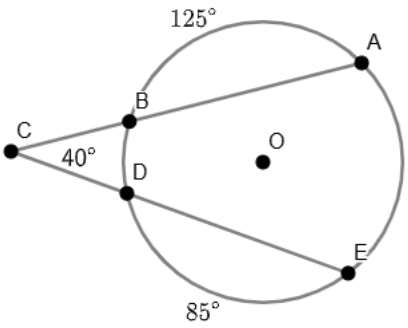
20. The diagonals of rhombus $ABCD$ as shown have lengths $AC = 30$ and $BD = 50$. Determine the perpendicular distance between $\overline{AB}$ and $\overline{DC}$.

(**A**) $\frac{29\sqrt{34}}{17}$ (**B**) $\frac{48\sqrt{34}}{17}$ (**C**) $\frac{75\sqrt{34}}{17}$

(**D**) $\frac{130\sqrt{34}}{17}$ (**E**) None of these



21. In the circle shown, $m\angle C = 40°$, $m\widehat{AB} = 125°$ and $m\widehat{ED} = 85°$. Determine $m\widehat{AE}$.



(**A**) 100° (**B**) 105° (**C**) 110° (**D**) 115° (**E**) None of these

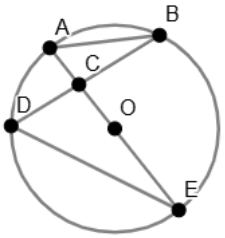
22. A fenced-in rectangular plot of land contains a rectangular pool surrounded by a lawn. If the pool is 40 meters by 16 meters and is exactly 2 meters from the fence at all points, find the area of the lawn.

(**A**) 214 m^2 (**B**) 240 m^2 (**C**) 266 m^2 (**D**) 282 m^2 (**E**) None of these

23. In $\triangle ABC$, $BC = 13$, $CA = 14$, and $AB = 15$. If D is a point on $\overline{CA}$ such that $\overline{BD}$ is perpendicular to $\overline{CA}$, then what is the length of $\overline{BD}$?

(**A**) 9 (**B**) 10 (**C**) 11 (**D**) 12 (**E**) None of these

24. In the circle shown, there are four chords such that two intersect at point C and chord $\overline{AE}$ is the diameter of the circle. If $AB = 6$, $BC = 8$, $CD = 4$, and $DE = 12$, then what is the area of the circle?



(**A**) 27π (**B**) 64π (**C**) 81π (**D**) 96π (**E**) None of these

25. A regular hexagon has sides 16 units long. Find the perimeter of the triangle formed by connecting alternate vertices.

(**A**) $32\sqrt{3}$ units (**B**) $48\sqrt{3}$ units (**C**) $64\sqrt{3}$ units (**D**) $80\sqrt{3}$ units (**E**) None of these

26. In right triangle $\triangle ABC$, where the right angle is at vertex C, the altitude from vertex C divides the hypotenuse into two segments, one of length 64 units and the other length 16 units. Determine the area of the triangle.

(**A**) 840 units^2 (**B**) 964 units^2 (**C**) 1088 units^2 (**D**) 1132 units^2 (**E**) None of these

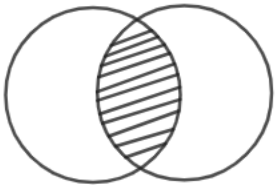
27. In a given triangle, one angle is twice as large as the other, and the third is 45° smaller than the sum of the other two. Find the measure of the smallest of the three angles.

(**A**) 35° (**B**) 37.5° (**C**) 40° (**D**) 42.5° (**E**) None of these

28. If the perimeters of a square and an equilateral triangle both equal 3, then find the positive difference of the area of the square and the area of the triangle?

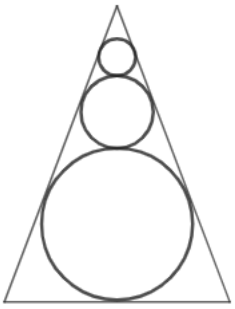
(**A**) $\frac{9-4\sqrt{3}}{16}$ (**B**) $\frac{6-2\sqrt{3}}{16}$ (**C**) $\frac{8\sqrt{3}-11}{16}$ (**D**) $\frac{9-2\sqrt{3}}{16}$ (**E**) None of these

29. Two identical circles intersect as shown. The area of the shaded region is equal to the sum of the areas of the two unshaded regions. Suppose the area of the shaded region is 64π. Determine the circumference of each circle.



(**A**) $3\sqrt{6}\pi$ (**B**) 8π (**C**) $8\sqrt{6}\pi$ (**D**) 10π (**E**) None of these

30. Three spheres are inscribed in a right circular cone as shown from the side view given in the figure. The radius of the spheres are 4, 6, and r. Find the volume of the largest sphere with radius r.



(**A**) 972π (**B**) 1084π (**C**) 1168π (**D**) 1446π (**E**) None of these

31. The ratio of the dimensions of a rectangular solid is $5:4:3$ and the ratio of this solid's volume to its surface area is $2:1$. What is the length of the solid's longest side?

(**A**) 14 (**B**) $15\frac{2}{3}$ (**C**) 16 (**D**) $18\frac{1}{4}$ (**E**) None of these

32. Find the distance between the three-dimensional points $(6, 4, -3)$ and $(2, -8, 3)$.

(**A**) 14 (**B**) $15\sqrt{2}$ (**C**) 16 (**D**) $12\sqrt{3}$ (**E**) None of these

33. Two equilateral triangles sharing an edge have a combined area of π. What is the square of the length of their shared edge?

(**A**) $\frac{\pi\sqrt{3}}{3}$ (**B**) $\frac{2\pi\sqrt{3}}{3}$ (**C**) $\frac{4\pi\sqrt{3}}{3}$ (**D**) $\frac{5\pi\sqrt{3}}{3}$ (**E**) None of these

34. In triangle $\triangle ABC$, let D be the intersection point of the bisector of angle $\angle ABC$ and the angle $\angle BCA$. If $m\angle CAB$ is $70°$, what is $m\angle CDB$?

(**A**) $105°$ (**B**) $110°$ (**C**) $125°$ (**D**) $130°$ (**E**) None of these

35. Let $AB = 2\sqrt{3}$ and $CD = 2$ be two chords on a circle with center O. The ratio of the distance from O to AB and the distance from O to CD is $1:\sqrt{3}$. Find the circumference of the circle.

(**A**) π (**B**) $\sqrt{2}\pi$ (**C**) 2π (**D**) $2\sqrt{2}\pi$ (**E**) None of these

36. Find the y-coordinate of the point on the y-axis that is equidistant from $(5, -5)$ and $(1, 1)$.

(**A**) -2 (**B**) -4 (**C**) 0 (**D**) 3 (**E**) None of these

37. The sides a, b, and c of a triangle satisfy $\sqrt{a} + \sqrt{b} = \sqrt{c}$. Which of the following best describes the triangle?

(**A**) Acute (**B**) Scalene (**C**) Isosceles (**D**) Equilateral (**E**) None of these

38. What is the volume of a right triangular prism whose height is 8 units and whose base has sides of lengths 5, 6, and 7 units?

(**A**) $36\sqrt{6}$ units^3 (**B**) $48\sqrt{6}$ units^3 (**C**) $64\sqrt{6}$ units^3 (**D**) $81\sqrt{6}$ units^3 (**E**) None of these

39. Rectangle $ABCD$ has the two sides $\overline{AD}$ and $\overline{BC}$ of length 18. Point F is on side $\overline{BC}$ and point E is on side $\overline{CD}$. Suppose $AB = AE$, $CE = CF$, and $FB = FE$. Find the length of $\overline{AB}$.

(**A**) $9\sqrt{2}$ (**B**) $9\sqrt{3}$ (**C**) $18\sqrt{2}$ (**D**) $18\sqrt{3}$ (**E**) None of these

40. A person walks around an equilateral triangle with side lengths of 4 ft. long. If a person walks fully around the triangle, keeping a constant distance of 4 ft. from the triangle at all times, how far in total have they walked?

(**A**) 16 ft (**B**) $(8 + 12\pi)$ ft (**C**) 20 ft (**D**) $(12 + 8\pi)$ ft (**E**) None of these

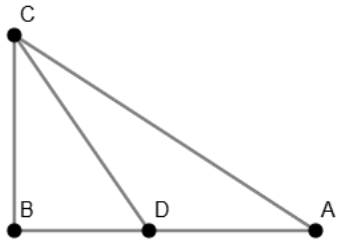
41. A circle is inscribed in an isosceles trapezoid which has bases measuring 8 and 18. Find the area between the circle and isosceles trapezoid.

(**A**) $144 - 28\pi$ (**B**) $121 - 16\pi$ (**C**) $172 - 49\pi$ (**D**) $156 - 36\pi$ (**E**) None of these

42. Suppose $m\angle ABC = 90°$, $m\angle CDB = 45°$, $m\angle CAB = 30°$, and $AD = 2$. Then BC equals:

(**A**) $\sqrt{3} + 1$ (**B**) $2\sqrt{3}$ (**C**) $2\sqrt{3} + 1$

(**D**) $\sqrt{3} + 2$ (**E**) None of these



43. Suppose that $\triangle ABC$ is a triangle inscribed in a circle whose center O lies on $\overline{AB}$. Let P be the point of intersection of the line $\overleftrightarrow{BC}$ and the line tangent to the circle at A. If $AP = AO = 1$, determine the length of $\overline{AC}$.

(**A**) $\frac{2\sqrt{5}}{5}$ units (**B**) $\frac{2\sqrt{3}}{3}$ units (**C**) $\frac{3\sqrt{2}}{2}$ units (**D**) $\frac{5\sqrt{2}}{2}$ units (**E**) None of these

44. Two different circles each with radius 1 lie completely within a square. Their centers lie along one of the diagonals of the square. Each circle is tangent to two sides of the square and the circles are tangent to each other. Find the area of the square.

(**A**) $6 + 4\sqrt{2}$ (**B**) $10 + 2\sqrt{2}$ (**C**) $4 + 6\sqrt{2}$ (**D**) $8 + 2\sqrt{2}$ (**E**) None of these

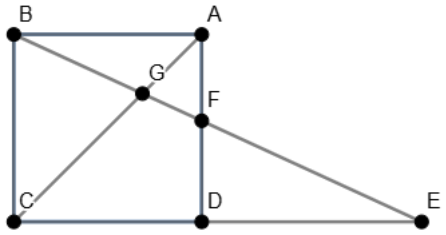
45. A Pythagorean triple is the set of three positive integers that satisfy $a^2 + b^2 = c^2$. If the smallest of the three is 21 and the difference of the other two is 3, find the area of a triangle that can be constructed with these lengths.

(**A**) 452 (**B**) 538 (**C**) 614 (**D**) 756 (**E**) None of these

46. An open rectangular box is to be constructed with single ply cardboard on the sides and double ply on the bottom. Single ply cardboard costs 10 cents per square foot and double ply costs is 15 cents per square foot. Find the cost of a box with square base and height twice its length if the volume is to be 54 cubic feet.

(**A**) \$8.55 (**B**) \$9.20 (**C**) \$9.55 (**D**) \$9.70 (**E**) None of these

47. In square $ABCD$, a line through B intersects the extension of $\overline{CD}$ at E, the side $\overline{AD}$ at F, and the diagonal $\overline{AC}$ at G. If $BG = 3$ and $GF = 1$, then what is the length of $\overline{BE}$?



(**A**) 4 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) None of these

48. A cube that is inside a sphere has its eight corners touching the sphere. What is the ratio of the volume of the sphere to the volume of the cube?

(**A**) $\frac{1}{2}\pi$ (**B**) $\frac{\sqrt{3}}{4}\pi$ (**C**) $\frac{\sqrt{3}}{2}\pi$ (**D**) $\frac{3}{2}\pi$ (**E**) None of these

49. $2K+1$ points are evenly spaced on a circle are numbered 0 through $2K$ in succession. Determine the measure of the angle formed at point K by connecting the point 0 to point K to point $2K$.

(**A**) $\frac{90}{2K+1}$ (**B**) $\frac{180}{2K+1}$ (**C**) $\frac{270}{2K+1}$ (**D**) $\frac{360}{2K+1}$ (**E**) None of these

50. An isosceles right triangular region of area 25 is cut from a corner of a rectangular region with sides of length $5\sqrt{2}$ and $5(1+\sqrt{2})$. What is the perimeter of the resulting trapezoid?

(**A**) $5+20\sqrt{2}$ (**B**) $10+10\sqrt{2}$ (**C**) $10+20\sqrt{2}$ (**D**) $20+10\sqrt{2}$ (**E**) None of these