Alabama Statewide Math Contest - Round 4 Division Two

University of Alabama Birmingham

April 6, 2024

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET		

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$ and $3 - 2\sqrt{3}$.

Round 4

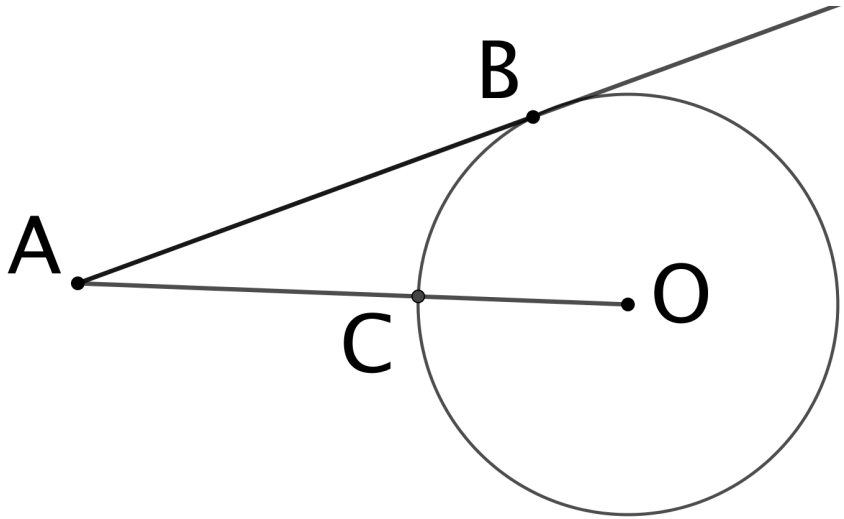
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

In the figure, points B and C lie on the circle centered at O such that C is on $\overline{AO}$ and $\overleftrightarrow{AB}$ is tangent to the circle. If $AB = 3\sqrt{5}$ and $AC = 5$, what is the radius of the circle?



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 2

Geometry Question # 2

Geometry Question # 2

RESET

A right circular cylinder and a sphere of the same radius both have a surface area of 98π. What is the height of the cylinder?

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $\frac{7\sqrt{2}}{2}$

Round 4

Algebra II

Algebra II Question # 3

Algebra II Question # 3

RESET

The quadratic function $f(x) = ax^2 + 5x + c$ has a maximum value of $y = 7$ at $x = 4$. What is the product ac?

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: $\frac{15}{8}$

Algebra II Question # 4

Algebra II Question # 4

RESET

What is the value of the product $i \cdot i^2 \cdots i^{10}$?

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: $-i$

Round 4

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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Find the solution x to the equation

$$\ln(2x-3)+\ln(x+4)=\ln(2x^2+11)$$

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

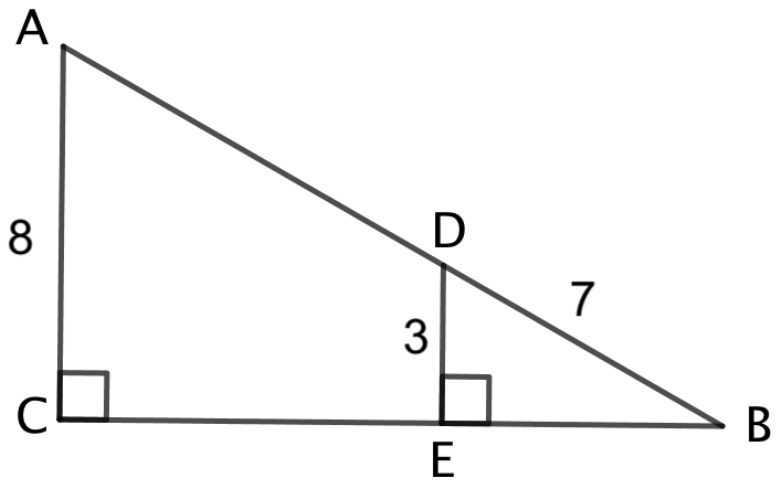
Answer: $\frac{23}{5}$

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

In the figure, points D and E lie on segments $\overline{AB}$ and $\overline{BC}$ respectively, with $\angle ACB$ and $\angle DEB$ both right angles. If $AC = 8$, $DE = 3$ and $BD = 7$, find $\sin(\angle BAC)$.



Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{2\sqrt{10}}{7}$

Round 4

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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What is the (2,3) entry in the matrix product below?

$$\begin{bmatrix} 5 & 8 \\ -9 & 1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 5 & 0 & 8 & -2 \\ 4 & 4 & -3 & 6 \end{bmatrix}$$

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: -75

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET		

What is the smallest value of x that is divisible by the numbers 1 through 8?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 840

Round 4

Team

Team Question # 9

Team Question # 9

RESET		

The vertical asymptote of $f(x) = \dfrac{-3x^2 + 2}{x - 1}$ crosses the slant asymptote of $f(x)$ at point (a, b). What is b?

Team Question # 9

Answer:

Team Question # 9

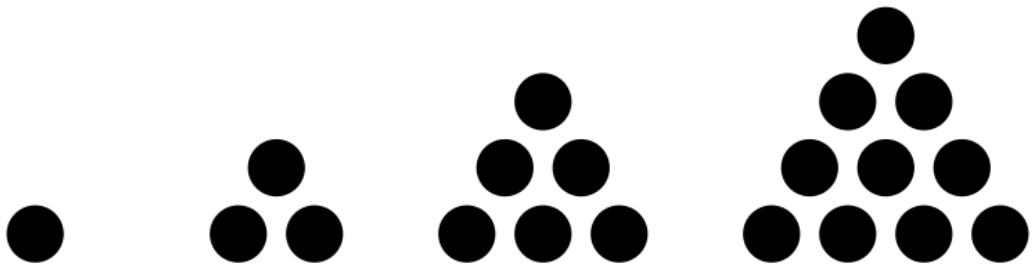
Answer: -6

Team Question # 10

Team Question # 10

RESET		

A triangular number counts objects that can be arranged in an equilateral triangle. The first four triangular numbers are 1, 3, 6, and 10 as pictured below:



What is the 15th triangular number?

Team Question # 10

Answer:

Team Question # 10

Answer: 120

End of Round 4