Alabama Statewide Math Contest - Round 3 Division Two

University of Alabama Birmingham

April 6, 2024

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET		

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$ and $3 - 2\sqrt{3}$.

Round 3

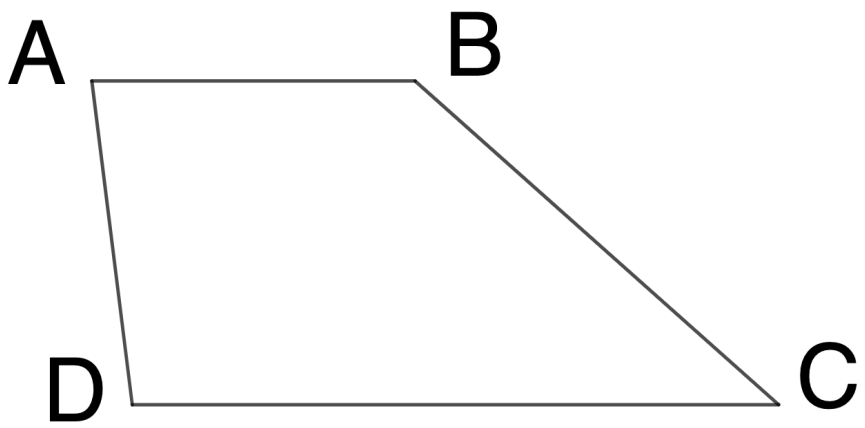
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

In $\square ABCD$, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, $AB = AD$, $BC = BD$, and $m\angle BAD = 80°$. What is the measure of angle $\angle BCD$, in degrees?



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 50

Geometry Question # 2

Geometry Question # 2

RESET

Let $\triangle ABC$ have $AB = 4$, $AC = 6$ and $BC = 8$. Given that the area of $\triangle ABC$ is $3\sqrt{15}$, find the length of the longest altitude.

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $\frac{3\sqrt{15}}{2}$

Round 3

Algebra II

Algebra II Question # 3

Algebra II Question # 3

RESET

Define an operation $\star$ on non-negative real numbers by $a \star b = a\sqrt{b}$. What value of x makes

$$(7 \star 4) + (x \star x) = 78?$$

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: 16

Algebra II Question # 4

Algebra II Question # 4

RESET

If $f(x) = x^2 + bx + c$ has a zero of $5i$ for b, c rational numbers, what is the sum of $b + c$?

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: 25

Round 3

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

How many distinct arrangements are there of the symbols

$$\star\star\star\triangle\triangle\sqcap\diamond$$

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 420

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Find the smallest positive solution, in radians, to the equation

$$2\sin^2\theta = 1 - \cos\theta$$

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{2\pi}{3}$

Round 3

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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What is the distance between the intersection points of the graphs of $y = x^2 + 2x$ and $y = 5 - 2x$?

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: $6\sqrt{5}$

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET		

An irregular pentagon $ABCDE$ has right angles at vertices A and C, and the angles at the remaining vertices congruent to each other. Additionally, we have $EA = AB = BC = CD = 4$. If the area of the pentagon is 24, what is the area of $\triangle BDE$?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 8

Round 3

Team

Team Question # 9

Team Question # 9

RESET

Find the sum of all values of x for which the three numbers of the set $\{5, 6, x\}$ have the same mean and median.
(Note: this notation does not mean we are assuming $x \geq 6$.)

Team Question # 9

Answer:

Team Question # 9

Answer: 16.5

Team Question # 10

Team Question # 10

RESET

A quadratic function $p(x)$ has a remainder of -3 when divided by $x+2$, a remainder of -3 when divided by $x+1$, and a y-intercept of 7. What is the x-coordinate of the vertex of this quadratic?

Team Question # 10

Answer:

Team Question # 10

Answer: $-\frac{3}{2}$

End of Round 3