Alabama Statewide Math Contest - Round 2 Division Two

University of Alabama Birmingham

April 6, 2024

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET		

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$ and $3 - 2\sqrt{3}$.

Round 2

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

Let $\triangle ABC$ be a triangle with D on side $\overline{AB}$ and E on side $\overline{AC}$ such that $\overleftrightarrow{DE}$ and $\overleftrightarrow{BC}$ are parallel. If $AD = x$, $AE = x + 3$, $BD = 3x + 4$ and $CE = 3x + 19$, what is the value of x?

Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 2

Geometry Question # 2

Geometry Question # 2

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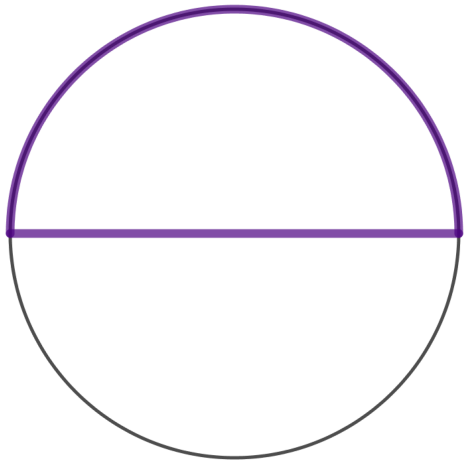
In the figure shown, the perimeter of the outlined semicircle, which includes the diameter of the circle, is $8 + 4\pi$. What is the area of the circle?



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 16π

Round 2

Algebra II

Algebra II Question # 3

Algebra II Question # 3

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Find all real solutions to the equation

$$\sqrt{3x-5} = 2 - \sqrt{x-1}$$

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: 2

Algebra II Question # 4

Algebra II Question # 4

RESET		

Solve for b in the system of equations

$$\begin{cases} 2^a 2^b &= 32 \\ 2^{-3a} 4^b &= 16 \end{cases}$$

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: $\frac{19}{5}$

Round 2

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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What is the sum of all values of a for which the graphs of $y = \frac{1}{2}(x-4)^2$ and $y = ax - 10$ intersect exactly once?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: -8

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET

Find the positive difference between the two real solutions to the equation

$$\frac{3}{y} - \frac{y}{5} = \frac{59}{10}$$

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{61}{2}$

Round 2

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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A trapezoid with an area of 92 in^2 has bases which differ in length by 5. If the difference of the squares of the lengths of the bases is 115, what is the height of the trapezoid?

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 8

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

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An arithmetic sequence begins $A_1 = 5$, $A_2 = 8$, $A_3 = 11$. What is the value of A_{2024}?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 6074

Round 2

Team

Team Question # 9

Team Question # 9

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If a is the area of a square with a diagonal of $3\sqrt{2}$ and b is the smallest integer solution to the inequality $7x - 2 \geq 10 - 3x$, find distance from the origin to the image of the point (a, b) after reflecting about the line $y = 3$.

Team Question # 9

Answer:

Team Question # 9

Answer: $\sqrt{97}$

Team Question # 10

Team Question # 10

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Find the sum of the x values where the graph of $f(x) = \dfrac{x+2}{x-2}$ intersects the graph of its inverse.

Team Question # 10

Answer:

Team Question # 10

Answer: 3

End of Round 2