Alabama Statewide Math Contest - Round 1 Division Two

University of Alabama Birmingham

April 6, 2024

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET		

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$ and $3 - 2\sqrt{3}$.

Round 1

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

What is the perimeter of an isosceles right triangle with a hypotenuse of 5?

Geometry Question # 1

Answer:

Geometry Question # 1

Answer: $5\sqrt{2} + 5$

Geometry Question # 2

Geometry Question # 2

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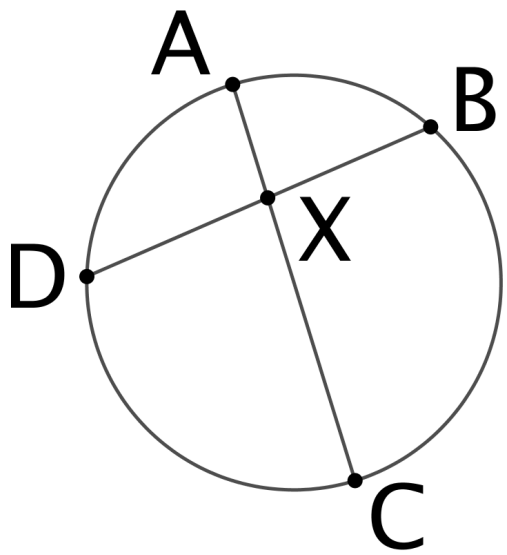
In the figure, points A, B, C, D lie on the circle of radius 10.5, and chords $\overline{AC}$ and $\overline{BD}$ cross at point X. If $\overline{AC}$ is a diameter, $AX = 6$, and DX is one unit longer than BX, what is the length of BD?



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 19

Round 1

Algebra II

Algebra II Question # 3

Algebra II Question # 3

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What is the sum of the solutions to the equation $|2x - 1| = |4x + 9|$?

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: $-\frac{19}{3}$

Algebra II Question # 4

Algebra II Question # 4

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A particular polynomial has roots at $x = -1$, $x = 0$ and $x = 5$, and a minimum value at $x \approx 3.5$. What is the smallest possible degree of this polynomial?

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: 4

Round 1

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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Find the solution to $(x^2 - x + 4)^{3/4} - x^{3/2} = 0$.

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 4

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

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A pile of cards has two red cards and three black cards. If you draw two cards at random without replacement, what is the probability they are the same color?

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{2}{5}$

Round 1

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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The distance from a point $(x, -3)$ to $(9, x)$ is $4\sqrt{5}$. What is the sum of all possible values of x?

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 6

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

The polynomial $f(x) = 2x^3 - 3x^2 - 17x - 12$ has a root at $x = -1$. Find the largest remaining root.

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 4

Round 1

Team

Team Question # 9

Team Question # 9

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Let r be the radius of the circle given by the equation $x^2 + y^2 - 4x - 6y - 12 = 0$, let m be the mean of the two roots of the function $y = x^2 - 7x + 9$, and let n be the number of solutions to the equation $\sin(2x) + 2\cos x = 0$ on $[0, 2\pi)$. Find the product rmn.

Team Question # 9

Answer:

Team Question # 9

Answer: 35

Team Question # 10

Team Question # 10

RESET

An integer is called square-free if its prime decomposition has no repeated factors. For example, $70 = 2 \times 5 \times 7$ is square free, but $75 = 3 \times 5 \times 5$ is not.

How many values of x are square-free numbers such that $10 \leq x \leq 30$?

Team Question # 10

Answer:

Team Question # 10

Answer: 13

End of Round 1