The 42nd Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

Written Round: February 25, 2023 at Regional Testing Sites

Ciphering Round: April 15, 2023 at University of North Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown, Auburn University at Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E. A calculator is NOT permitted.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.
- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Editing by Miranda Bowie and Ashley Johnson, The University of North Alabama
Printing by The University of Alabama at Birmingham

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. Given the points A at $(8, k)$, B at $(5, 2)$ and C at $(10, -3)$, find the value of k so that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

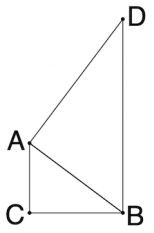
(**A**) -1 (**B**) 1 (**C**) -5 (**D**) 5 (**E**) None of these

2. A cone has a height six times its radius. The volume of the cone is 128π. What is the diameter of the cone?

(**A**) 4 (**B**) $4\sqrt[3]{9}$ (**C**) 8 (**D**) $8\sqrt[3]{9}$ (**E**) None of these

3. Suppose $\triangle ABC$ is a right triangle, with legs $AC = 3$ and $BC = 4$. Let D be a point such that $\overleftrightarrow{DB}$ is perpendicular to $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$ is perpendicular to $\overleftrightarrow{AB}$. Find the perimeter of $\triangle ABD$.

(**A**) 20 (**B**) 24 (**C**) 27 (**D**) 30 (**E**) None of these



4. A convex polygon has 14 diagonals. How many sides does this polygon have?

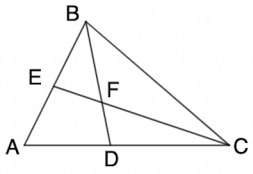
(**A**) 6 (**B**) 7 (**C**) 8 (**D**) 9 (**E**) None of these

5. Let $ABCDEF$ be a regular hexagon, with a side length of 5. What is the area of quadrilateral $ABDE$?

(**A**) 25 (**B**) $25\sqrt{2}$ (**C**) $25\sqrt{3}$ (**D**) 50 (**E**) None of these

6. In $\triangle ABC$, rays $\overrightarrow{BF}$ and $\overrightarrow{CF}$ are angle bisectors of angles $\angle ABC$ and $\angle ACB$ respectively, intersecting the opposite sides at points D and E, respectively. Given that $m\angle BFC$ is 110°, find $m\angle BAC$.

(**A**) 40° (**B**) 50° (**C**) 60° (**D**) 70° (**E**) None of these



7. In a circle, two chords $\overline{AB}$ and $\overline{CD}$ intersect at point E. If $AE = 2.5$, $BE = 6.4$ and $CE = DE$, what is the length of $\overline{CD}$?

(**A**) 4 (**B**) 8 (**C**) 16 (**D**) 32 (**E**) None of these

8. In a regular dodecagon, what is the difference between the measure of an interior angle and the measure of an exterior angle, in degrees?

(**A**) 108 (**B**) 120 (**C**) 144 (**D**) 150 (**E**) None of these

9. The volume of a sphere of radius 8 cm is equal to the volume of a right square based pyramid, with a base side length of 32. What is the height of the pyramid?

(**A**) $\frac{\pi}{4}$ (**B**) $\frac{3\pi}{4}$ (**C**) π (**D**) 2π (**E**) None of these

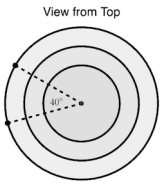
10. A point (x, y) is 5 units from the origin on the line $y = -2x$. Find x^2y^2.

(**A**) 100 (**B**) 400 (**C**) 500 (**D**) 2500 (**E**) None of these

11. What is the length in cm of a 144° arc in a circle with a diameter of 60 cm?

(**A**) 12π (**B**) 24π (**C**) 36π (**D**) 48π (**E**) None of these

12. A three-layer birthday cake has a bottom layer with a 10 inch diameter, a middle layer with an 8 inch diameter and a top layer with a 6 inch diameter. One slice has been removed, cut with two radii meeting at a 40° angle. If each layer is 2" tall, how much birthday cake is remaining, in cubic inches?



(**A**) $\frac{200\pi}{9}$ (**B**) $\frac{400\pi}{9}$ (**C**) $\frac{800\pi}{9}$ (**D**) $\frac{1600\pi}{9}$ (**E**) None of these

13. A particular rectangle has an area of 48 u^2 and a perimeter of $28u$. Determine the length of one of its diagonals in units.

(**A**) $4\sqrt{181}$ (**B**) $4\sqrt{193}$ (**C**) $6\sqrt{167}$ (**D**) $6\sqrt{177}$ (**E**) None of these

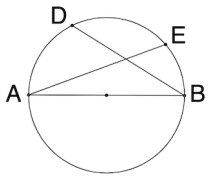
14. At night, a man who is 6 feet tall stands 10 feet away from a lamppost. If the man's shadow measures 4 feet, how far above the ground is the light bulb in the lamppost?

(**A**) 12 feet (**B**) 15 feet (**C**) 18 feet (**D**) 21 feet (**E**) None of these

15. A convex polygon has interior angles that measure, in degrees, $2x$, $6x+5$, $8x-5$, $7x$ and $5x-20$. Find the difference in the measure of the largest angle of this polygon and its smallest.

(**A**) 40° (**B**) 55° (**C**) 105° (**D**) 115° (**E**) None of these

16. In the figure shown, $\overline{AB}$ is a diameter of the circle. If $m\angle EAB = 25°$ and $m\angle ABD = 35°$, what is the measure of arc $\overset{\frown}{DE}$?



(**A**) 20° (**B**) 30° (**C**) 60° (**D**) 120° (**E**) None of these

17. In triangle ABC, $m\angle BAC$ is three times $m\angle ABC$ and $m\angle ACB$ is 20° larger than the sum of $m\angle BAC$ and $m\angle ABC$. What is the measure of the largest angle?

(**A**) 75° (**B**) 80° (**C**) 90° (**D**) 100° (**E**) None of these

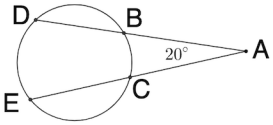
18. In right triangle ABE, $\angle ABE$ is a right angle with $AB = 14$. Let D be a point on $\overline{BE}$ and C a point on $\overline{AE}$ such that $\overline{CD} \perp \overline{BE}$, $BD = 12$, and $DE = 16$. What is the value of $\sin\angle DBC$?

(**A**) $\frac{\sqrt{13}}{13}$ (**B**) $\frac{2\sqrt{13}}{13}$ (**C**) $\frac{3\sqrt{13}}{13}$ (**D**) $\frac{4\sqrt{13}}{13}$ (**E**) None of these

19. A triangle $\triangle ADC$ is inscribed in a circle in which $\overline{AC}$ is a diameter. Let B be a point on $\overline{AC}$ such that $\overline{DB} \perp \overline{AC}$, $AB = 9$ and $BD = 12$. Find the area of $\triangle DBC$.

(**A**) 30 (**B**) 54 (**C**) 96 (**D**) 150 (**E**) None of these

20. In the figure shown, points D, B, E, and C are on the circle, and rays $\overrightarrow{DB}$ and $\overrightarrow{EC}$ meet at point A, with $m\angle BAC = 20°$. If $m\overset{\frown}{BD} = m\overset{\frown}{DE} = m\overset{\frown}{CE}$, find the measure of angle $\angle DBE$.



(**A**) 20° (**B**) 40° (**C**) 50° (**D**) 100° (**E**) None of these

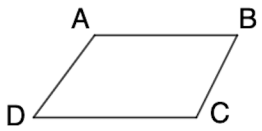
21. Let $\square ABCD$ be a rectangle with $AB = 10$ and $BC = 8$. Suppose E is the midpoint of side $\overline{AB}$ and F is the midpoint of side $\overline{BC}$. Let G be a point on $\overline{CD}$ so that the area of $\triangle EFG$ is 18 square units. What is the length of the longest side of $\triangle EFG$?

(**A**) $2\sqrt{17}$ (**B**) $\sqrt{41}$ (**C**) $\sqrt{65}$ (**D**) $\sqrt{73}$ (**E**) None of these

22. Consider points A, B, C, and D on a circle such that ray $\overrightarrow{BA}$ intersects ray $\overrightarrow{DC}$ at a point P external to the circle. If $PB = 27$ in, $AB = 21$ in, and $PC = CD$, find the length of PD.

(**A**) 9 in (**B**) $9\sqrt{2}$ in (**C**) 18 in (**D**) $18\sqrt{2}$ in (**E**) None of these

23. Let $\square ABCD$ be a quadrilateral with $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$, $m\angle ADC = 45°$, $m\angle BCD = 120°$, $AD = 12\sqrt{2}$ and $CD = 27$. Find the length of $\overline{AB}$.



(**A**) $15 + 4\sqrt{2}$ (**B**) $15 + 12\sqrt{2}$ (**C**) $15 + 2\sqrt{3}$

(**D**) $15 + 4\sqrt{3}$ (**E**) None of these

24. Let $\square ABCD$ be a trapezoid, with $\angle ABC$ and $\angle BCD$ both right angles. If $AB = 84$cm, $BC = 40$cm, and the area of the trapezoid is 3180 cm^2, what is the perimeter of $\square ABCD$ in cm?

(**A**) 240 (**B**) 248 (**C**) 310 (**D**) 318 (**E**) None of these

25. What is the area of the circle defined by the equation $x^2 + 6x + y^2 - 4y - 3 = 0$?

(**A**) 4π (**B**) 16π (**C**) 64π (**D**) 256π (**E**) None of these

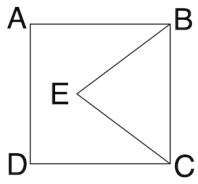
26. The volume of two cubes differ by $259u^3$. If the edges of one cube are each $4u$ greater than the edges of the other, what is the volume of the smaller cube?

(**A**) $\frac{125}{8}u^3$ (**B**) $\frac{2197}{8}u^3$ (**C**) $27u^3$ (**D**) $343u^3$ (**E**) None of these

27. Let O_1 and O_2 be concentric circles with the radius of O_2 larger than the radius of O_1. The length of a cord of O_2, which is tangent to O_1 is 22 units. Find the area of the region inside O_2 but outside O_1.

(**A**) 25π (**B**) 104π (**C**) 121π (**D**) 220π (**E**) None of these

28. Given square $\square ABCD$ and equilateral triangle $\triangle BCE$, find the measure of $\angle AED$.



(**A**) 90° (**B**) 120° (**C**) 135° (**D**) 150° (**E**) None of these

29. If (a, b) and (c, d) are the two intersection points of the line $y = 2x + 1$ and the circle centered at $(1, -1)$ with radius 4, what is $a + b + c + d$?

(**A**) $-\frac{8}{5}$ (**B**) $\frac{18}{5}$ (**C**) $\frac{4}{11}$ (**D**) $\frac{40}{11}$ (**E**) None of these

30. In a circle, a central angle with a measurement of 4.5 radians intercepts an arc of 108 inches. Find the area of the circle in square inches.

(**A**) 441π (**B**) 576π (**C**) 676π (**D**) 729π (**E**) None of these

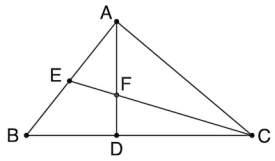
31. The circumcenter of a triangle is an interior point of the triangle if the triangle is:

(**A**) Acute (**B**) Right (**C**) Obtuse (**D**) All of these (**E**) None of these

32. Find the perimeter of the regular polygon with a side length of 16 and exterior angles measuring $40°$.

(**A**) 128 (**B**) 144 (**C**) 160 (**D**) 192 (**E**) None of these

33. In triangle $\triangle ABC$, $\angle BAC$ is a right angle, $BC = 8$, and $AC = 4\sqrt{3}$. Let $\overline{CE}$ be the median to side $\overline{AB}$ and $\overline{AD}$ an altitude to side $\overline{BC}$. Segments $\overline{CE}$ and $\overline{AD}$ intersect at point F. What is the length of $\overline{AF}$?



(**A**) $\sqrt{3}$ (**B**) $\frac{4\sqrt{3}}{3}$ (**C**) $\frac{6\sqrt{3}}{5}$ (**D**) $\frac{8\sqrt{3}}{7}$ (**E**) None of these

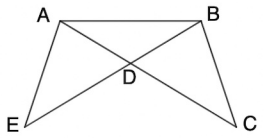
34. An isosceles trapezoid $ABCD$ with bases of length $AB = 36$ and $CD = 64$ has an inscribed circle. What is the diameter of the inscribed circle?

(**A**) 48 (**B**) 50 (**C**) $8\sqrt{34}$ (**D**) $9\sqrt{30}$ (**E**) None of these

35. How many lines of reflection symmetry does a regular 20-sided polygon have?

(**A**) 10 (**B**) 20 (**C**) 40 (**D**) 80 (**E**) None of these

36. In the figure, $AE = BC$, $AD = BD$, $DE = CD$. Let F be a point on $\overline{EC}$ such that $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ and let G be the point on $\overline{DC}$ where $\overline{DC}$ intersects $\overline{BF}$. Which of the following must be true?



I. $\square ABFE$ is a parallelogram
II. $\triangle BGD \sim \triangle EAD$
III. $\overrightarrow{BF}$ bisects $\angle EBC$

(**A**) I only (**B**) II only (**C**) I, II only (**D**) II, III only (**E**) I, II, III

37. An equilateral triangle with a side length of $2\sqrt{3}$ units is inscribed in a circle. What is the area of the circle?

(**A**) 3π (**B**) 4π (**C**) 9π (**D**) 12π (**E**) None of these

38. What is the slope of the line that bisects the acute angle given by the lines $y = 0$ and $y = x$?

(**A**) $\frac{1}{2}$ (**B**) $\frac{\sqrt{2}}{2}$ (**C**) $\frac{\sqrt{2}}{3}$ (**D**) $\sqrt{2} - 1$ (**E**) $\sqrt{2}$

39. Which of the following is not a Pythagorean triple?

(**A**) $(19, 180, 181)$ (**B**) $(36, 323, 325)$ (**C**) $(40, 198, 202)$ (**D**) $(47, 143, 151)$ (**E**) $(51, 140, 149)$

40. If $ABCDEFGH$ is a regular octagon, find $m\angle ACD$.

(**A**) $75°$ (**B**) $90°$ (**C**) $120°$ (**D**) $135°$ (**E**) None of these

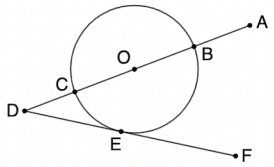
41. The point $(2, -3)$ goes to the point $(4, -5)$ after a dilation centered at point (a, b) with scale factor $k = \frac{5}{2}$. What is $a + b$?

(**A**) $\frac{2}{3}$ (**B**) $-\frac{2}{3}$ (**C**) 1 (**D**) -1 (**E**) None of these

42. Let $\overline{AB}$ be a diameter of the circle given by $x^2 + y^2 + 5x - 8y + 19 = 0$. What is the midpoint of segment $\overline{AB}$?

(**A**) $\left(-\frac{5}{2}, 4\right)$ (**B**) $\left(\frac{5}{2}, 4\right)$ (**C**) $\left(-\frac{5}{2}, -4\right)$ (**D**) $\left(\frac{5}{2}, -4\right)$ (**E**) None of these

43. In the figure, the circle is centered at O, $\overline{AD}$ is a secant passing through O and intersecting the circle at points B and C, $\overline{DF}$ is tangent to the circle at point E, and $\overline{BF}$ is perpendicular to $\overline{AD}$. If $m\widehat{BE} = 120°$, and the diameter of the circle is 10, find the length of $\overline{EF}$.



(**A**) $5\sqrt{3}$ (**B**) 10 (**C**) $5\sqrt{3} + 5$ (**D**) 15 (**E**) None of these

44. Quadrilateral $ABCD$ is inscribed in a circle with $m\angle BCD = 70°$. Find the measure of angle $\angle ABC$.

(**A**) 20° (**B**) 35° (**C**) 70° (**D**) 110° (**E**) Cannot be determined

45. What are the coordinates of the centroid of the triangle with vertices $(8, -18)$, $(12, -6)$, $(-3, 14)$?

(**A**) $\left(\frac{17}{3}, -\frac{10}{3}\right)$ (**B**) $\left(\frac{9}{2}, -2\right)$ (**C**) $\left(\frac{17}{6}, -\frac{5}{3}\right)$ (**D**) $\left(\frac{3}{2}, -\frac{2}{3}\right)$ (**E**) None of these

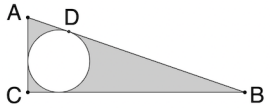
46. In right triangle $\triangle ABC$, with right angle at vertex C, arcs of a circle are drawn. The first arc is centered at A with a radius of 8, intersecting $\overline{AB}$ at a point D, and the second is centered at B with a radius of 15, intersecting $\overline{AB}$ at a point E. Point C lies on both arcs. Find the length of $\overline{DE}$.

(**A**) 4 (**B**) 5 (**C**) 6 (**D**) 7 (**E**) None of these

47. Let $\square ABCD$ be a rhombus and suppose $m\angle ADB = 22°$. Find $m\angle BAD$.

(**A**) 158° (**B**) 136° (**C**) 44° (**D**) 22° (**E**) None of these

48. Let $\triangle ABC$ be right triangle with right angle at vertex C, and let a circle be inscribed in $\triangle ABC$ with a point of tangency to side $\overline{AB}$ at D. If $AD = 7$ units, and the radius of the circle is 5 units, find the area of the shaded region, in units squared, rounded to the nearest integer.



(**A**) 131 (**B**) 135 (**C**) 139 (**D**) 143 (**E**) None of these

49. Let $\triangle ABC$ and $\triangle DEF$ be two triangles with $\angle BAC \cong \angle EDF$, $AB = DE$ and $BC = EF$. Which of the following additional assumptions would make $\triangle ABC \cong \triangle DEF$?

I. $\angle BAC$ is an acute angle II. $\angle BAC$ is a right angle III. $\angle BAC$ is an obtuse angle

(**A**) II only (**B**) III only (**C**) I and II only (**D**) II and III only (**E**) None of these

50. A cube has a volume of 343 cubic inches. What is the surface area of its circumscribed sphere?

(**A**) 98π (**B**) 147π (**C**) 392π (**D**) 588π (**E**) None of these