The 41st Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

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GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.
- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. The length of a rectangle exceeds its width by 3 feet. If the perimeter of the rectangle is 46 feet, what is the length, in feet?

(**A**) 7 (**B**) 9 (**C**) 11 (**D**) 13 (**E**) None of these

2. Find the midpoint of the x and y intercepts of the line $y = 2x - 4$.

(**A**) $(-2, 1)$ (**B**) $(-1, 2)$ (**C**) $(1, -2)$ (**D**) $(2, -1)$ (**E**) None of these

3. Two sides of a triangle measure 8 inches and 12 inches. If the altitude to the 8 inch side is 6 inches, what is the altitude to the 12 inch side?

(**A**) 4 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) None of these

4. Find the measure of an angle such that the sum of the measures of its supplement and its complement is $112°$.

(**A**) $68°$ (**B**) $73°$ (**C**) $79°$ (**D**) $84°$ (**E**) None of these

5. An isosceles triangle has two sides of length 10 and one of length 8. What is the area of the triangle?

(**A**) 40 (**B**) 80 (**C**) $4\sqrt{21}$ (**D**) $8\sqrt{21}$ (**E**) None of these

6. Suppose $\triangle ABD$ is a triangle, and let C be a point on side $\overline{BD}$ such that $m\angle CAD = m\angle DBA$. If $AB = 14$, $AD = 9$ and $CD = 4$, find BC.

(**A**) 9 (**B**) 12.25 (**C**) 14 (**D**) 16.25 (**E**) None of these

7. If the radius of a circle is decreased by 10%, by what percent is the area of the circle decreased?

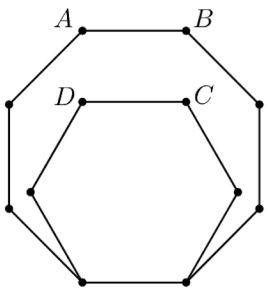
(**A**) 10% (**B**) 11% (**C**) 19% (**D**) 20% (**E**) None of these

8. The interior angles of a pentagon measure x, $x + 20$, $x + 40$, $x + 60$, and $x + 80$, each in degrees. Find the measure of the largest angle, in degrees.

(**A**) 104 (**B**) 112 (**C**) 140 (**D**) 148 (**E**) None of these

9. A regular hexagon of side length 4 is embedded in a regular octagon of side length 4, as shown. What is the area of quadrilateral $ABCD$?

(**A**) $16\sqrt{2}$ (**B**) $16\sqrt{3}$ (**C**) $16 + 16\sqrt{2} - 16\sqrt{3}$
(**D**) $16 + 16\sqrt{3} - 16\sqrt{2}$ (**E**) None of these



10. What is the perimeter of an equilateral triangle whose area is $16\sqrt{3}$?

(**A**) 12 (**B**) 24 (**C**) $12\sqrt{2}$ (**D**) $24\sqrt{2}$ (**E**) None of these

11. A cube has a surface area of 150 square units. What is its volume in cubic units?

(**A**) 100 (**B**) 125 (**C**) 150 (**D**) 175 (**E**) None of these

12. If a boat travels due North for 10 miles, then due East for 25 miles, how far is the boat from its starting point, rounded to the nearest mile?

(**A**) 26 miles (**B**) 27 miles (**C**) 28 miles (**D**) 29 miles (**E**) None of these

13. If a rhombus $ABCD$ has sides of length 10, and diagonal AC measures 12, what is the area of the rhombus?

(**A**) 60 (**B**) 96 (**C**) 100 (**D**) 120 (**E**) None of these

14. Which of the following is also a point on the line connecting points $(6, 12)$ and $(0, -6)$?

(**A**) $(-2, 0)$ (**B**) $(1, -5)$ (**C**) $(2, 8)$ (**D**) $(3, 3)$ (**E**) None of these

15. Four distinct points A, B, C, and D lie clockwise in that order on a circle. Chords $\overline{AB}$ and $\overline{DC}$ can be extended at B and C, respectively, to meet at point P, which is external to the circle. If $AB = 10$, $CD = 7$, $BP = 8$ and $m\angle APD = 60°$, find the area of the circle.

(**A**) $\frac{243}{4}\pi$ (**B**) $\frac{97}{2}\pi$ (**C**) 73π (**D**) 81π (**E**) None of these

16. In a quadrilateral ABCD, $\angle ABC$ and $\angle DAC$ are right angles, and $AB = 4$, $BC = 3$ and $AD = 12$. What is the area of $ABCD$?

(**A**) 30 (**B**) 36 (**C**) 72 (**D**) 144 (**E**) None of these

17. In a circle, two chords $\overline{AB}$ and $\overline{DC}$ intersect at point F. If $AF = 10, FB = 9$ and $FC = DF$, what is the length of $\overline{DC}$?

(**A**) $3\sqrt{10}$ (**B**) $6\sqrt{10}$ (**C**) $2\sqrt{15}$ (**D**) $4\sqrt{15}$ (**E**) None of these

18. A metal strip is shaped into a right triangle in such a way that it has its three sides in arithmetic progression. If the length of the shorter leg is 9, find the length of the hypotenuse.

(**A**) 11 (**B**) 13 (**C**) 15 (**D**) 17 (**E**) None of these

19. In triangle ABC, the measure of angle C is three times the measure of angle A and the measure of angle B is twice the measure of angle A. What is the ratio of the length of side $\overline{AB}$ to side $\overline{BC}$?

(**A**) 1:3 (**B**) 1:2 (**C**) 2:1 (**D**) 3:1 (**E**) None of these

20. The area of a rectangle is 10 and the length of its diagonal is 5. Find the difference of the square of the longer side and the square of the shorter side.

(**A**) 7 (**B**) 15 (**C**) 21 (**D**) 28 (**E**) None of these

21. What is the area of the triangle formed by the inequalities $3x + 2y \leq 9$, $x \geq 0$ and $y \geq 0$?

(**A**) $\frac{27}{4}$ (**B**) $\frac{9}{4}$ (**C**) $\frac{27}{2}$ (**D**) $\frac{9}{2}$ (**E**) None of these

22. In triangle ABC, the interior angle at vertex B measures 72°, and the exterior angle at vertex A measures 145°. What is the measure of the interior angle at vertex C, in degrees?

(**A**) 37 (**B**) 45 (**C**) 54 (**D**) 73 (**E**) None of these

23. A quadrilateral $ABCD$ is inscribed in a circle. If the measure of angle $\angle BAD$ is $32°$, then the measure of angle $\angle BCD$ is

(**A**) $32°$ (**B**) $58°$ (**C**) $122°$ (**D**) $148°$ (**E**) Not enough information

24. Suppose a circle of radius 6 has a central angle which intersects the circle at points A and B. If O is the center of the circle, and the intercepted arc $\widehat{AB}$ has length 4π, what is the measure of angle AOB in **radians**.

(**A**) $\frac{\pi}{3}$ (**B**) $\frac{\pi}{2}$ (**C**) $\frac{2\pi}{3}$ (**D**) $\frac{3\pi}{4}$ (**E**) None of these

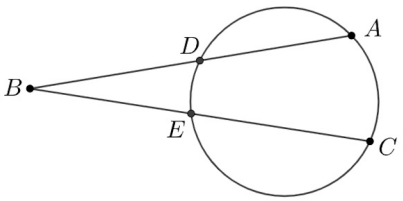
25. A cone has a height equal to six times its base radius. If the volume of the cone is 1000π, what is the base radius of the cone?

(**A**) 5 (**B**) $5\sqrt[3]{4}$ (**C**) 10 (**D**) $10\sqrt[3]{4}$ (**E**) None of these

26. In the figure shown, the measure of angle $\angle DBE$ is $38°$, and the measure of the minor arc $\widehat{DE}$ is $40°$. What is the measure of minor arc $\widehat{AC}$?

(**A**) $36°$ (**B**) $39°$ (**C**) $78°$

(**D**) $116°$ (**E**) None of these



27. Let AM be a median of triangle ABC with $AM = MB$, and $m\angle ABC = 40°$. What is the correct inequality representing the side lengths of the triangle?

(**A**) $AB > AC > BC$ (**B**) $BC > AC > AB$

(**C**) $BC > AB > AC$ (**D**) $AC > BC > AB$ (**E**) None of these

28. Consider a convex, but not necessarily regular, octagon. What is the ratio of the sum of its exterior angles to the sum of its interior angles?

(**A**) 1 : 4 (**B**) 1 : 3 (**C**) 1 : 2 (**D**) 1 : 1 (**E**) Not enough information to determine

29. A rectangular flower bed measures 6 yards by 4 yards. It is surrounded on all sides by a gravel path of uniform width such that the exterior of the path forms a rectangle. If the area of the gravel path equals that of the flower bed, what is the perimeter of the exterior of the gravel path?

(**A**) 24 yards (**B**) 26 yards (**C**) 28 yards (**D**) 30 yards (**E**) None of these

30. Find the radius in units of a sphere for which its volume in cubic units is equal to twice its surface area in square units.

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) None of these

31. A rectangular prism has its side, front, and bottom faces with area 3, 2, and 24 square inches, respectively. What is the volume of this prism in cubic inches?

(**A**) 12 (**B**) 48 (**C**) 72 (**D**) 144 (**E**) None of these

32. If the degree measure of an angle and its supplement are in the ratio of 2:7, what is the measure of the complement of the angle, in degrees?

(**A**) 40 (**B**) 50 (**C**) 60 (**D**) 70 (**E**) None of these

33. Which of the following regular polygons can be used to tile the plane? That is, which of the following regular polygons can, using only congruent copies of themselves, cover the plane with no gaps or overlaps?

I. Triangles II. Pentagons III. Hexagons IV. Octagons

(**A**) I, III, and IV (**B**) I and III (**C**) I, II, III, and IV (**D**) II and IV (**E**) III and IV

34. The area of a square is 116 square units. What is the largest possible area of a circle inscribed in this square, in square units?

(**A**) 29π (**B**) 58π (**C**) 116π (**D**) 232π (**E**) None of these

35. In triangle ABC, let D be the midpoint of side $\overline{AB}$ and E be the midpoint of side $\overline{AC}$. If $BC = 9x+2$ and $DE = 5x - 1$, find the length of $\overline{BC}$.

(**A**) 20 (**B**) 29 (**C**) 38 (**D**) 47 (**E**) None of these

36. Seven distinct points lie on a circle. How many different chords can be made using two of these seven points?

(**A**) 14 (**B**) 21 (**C**) 42 (**D**) 49 (**E**) None of these

37. Find the height of a square pyramid formed using four equilateral triangles whose sides all have length 2.

(**A**) $\sqrt{2}$ (**B**) $\sqrt{3}$ (**C**) $\sqrt{5}$ (**D**) $\sqrt{6}$ (**E**) None of these

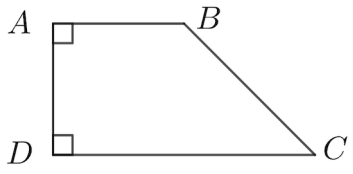
38. Triangle ABC and triangle DEF are similar, with the perimeter of $\triangle DEF$ equal to three times that of $\triangle ABC$. What is the ratio of the area of $\triangle DEF$ to the area of $\triangle ABC$?

(**A**) 3 : 1 (**B**) 6 : 1 (**C**) 9 : 1 (**D**) 12 : 1 (**E**) None of these

39. Two circles of the same radius, centered at points A and B, respectively, are externally tangent. The length of a tangent from point A to the circle centered at B is $3\sqrt{5}$. What is the area of either circle?

(**A**) 9π (**B**) 15π (**C**) 18π (**D**) 45π (**E**) None of these

40. In the figure provided, trapezoid $ABCD$ has right angles at vertices D and A, and $m\angle ABC = 120°$. If $AB = BC$, and $AC = 12$, what is the area of trapezoid $ABCD$?



(**A**) $24\sqrt{3}$ (**B**) $24\sqrt{3} + 24$ (**C**) $30\sqrt{3}$

(**D**) $30\sqrt{3} + 30$ (**E**) Not enough information

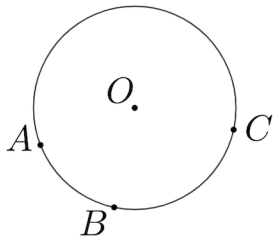
41. A line through the points $(m, -9)$ and $(7, m)$ has slope m. Find the value of m.

(**A**) -3 (**B**) -2 (**C**) 2 (**D**) 3 (**E**) None of these

42. The roots of the equation $x - 10 = -\dfrac{24}{x}$ represent two sides of a scalene triangle. Which of the following would be a valid third side to make the triangle an acute triangle?

(**A**) 1 (**B**) 3 (**C**) 9 (**D**) 11 (**E**) None of these

43. In the figure shown, point O is the center of the circle and A, B and C are three points on the circle. Suppose that $OA = AB = 2$, and angle $\angle OAC$ measures $10°$. Find the measure of minor arc $\overset{\frown}{BC}$ in degrees.



(**A**) 50 (**B**) 70 (**C**) 80 (**D**) 100 (**E**) None of these

44. Find the radius in units of the base of a right circular cylinder with a volume of 57π cubic units, and a height of 12 units.

(**A**) $\sqrt{14}$ (**B**) $\dfrac{\sqrt{14}}{2}$ (**C**) $\sqrt{19}$ (**D**) $\dfrac{\sqrt{19}}{2}$ (**E**) None of these

45. A circle with a center at $(6, 5)$ is tangent to a line at the point $(2, 7)$. Find the equation of the line tangent to the circle.

(**A**) $y = 2x + 3$ (**B**) $y = -x + 9$ (**C**) $y = -\dfrac{1}{2}x + 6$ (**D**) $y = x + 5$ (**E**) None of these

46. A cube is inscribed in a sphere. If the radius of the sphere is 1, what is the volume of the cube?

(**A**) $2\sqrt{2}$ (**B**) $\dfrac{\sqrt{2}}{4}$ (**C**) $\dfrac{8\sqrt{3}}{9}$ (**D**) $\dfrac{\sqrt{3}}{9}$ (**E**) None of these

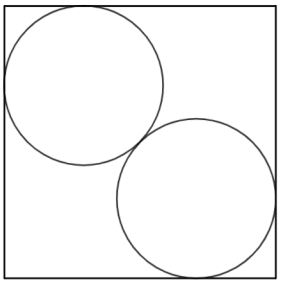
47. Given two intersecting circles $(x-2)^2 + (y-1)^2 = 4$ and $(x-3)^2 + (y-4)^2 = 9$, what is the equation of the line passing through the points of intersection?

(**A**) $2x + 6y = 15$ (**B**) $2x - 6y = 13$ (**C**) $6x - 2y = 11$ (**D**) $6x + 2y = 17$ (**E**) None of these

48. A wheel with a fixed center has a diameter of 18 inches. What is the number of revolutions of the wheel required for a fixed point on the wheel to travel exactly one mile? (Recall that there are 12 inches in one foot, and 5280 feet in one mile).

(**A**) $\dfrac{1760}{\pi}$ (**B**) $\dfrac{3520}{\pi}$ (**C**) $\dfrac{5280}{\pi}$ (**D**) $\dfrac{7040}{\pi}$ (**E**) None of these

49. Two identical circles are placed into a square in such a way that they are tangent to each other at a single point, and each circle is tangent to the square at two points, as shown. If the radius of each circle is 1, what is the area of the square?



(**A**) $\dfrac{25}{2}$ (**B**) $\dfrac{49}{4}$ (**C**) $3 + 2\sqrt{2}$ (**D**) $6 + 4\sqrt{2}$ (**E**) None of these

50. An equilateral triangle is inscribed in a circle of radius 4. What is the perimeter of the triangle?

(**A**) $12\sqrt{3}$ (**B**) $12\sqrt{2}$ (**C**) $8\sqrt{3}$ (**D**) $8\sqrt{2}$ (**E**) None of these