Written Round: April 9 − 11, 2021 at your school

GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

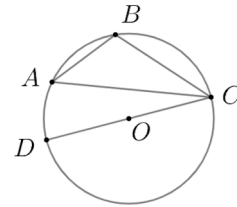**Where can I work?** What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

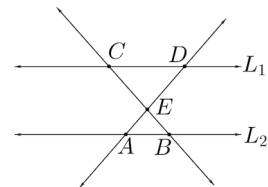**A Mathematics Major isn't just for those wanting to be Mathematicians!**

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. If the graphs of $4y + 2x + 6 = 0$ and $6y + ax + 4 = 0$ intersect at a right angle, what is the value of a?

   (**A**) $\boxed{-12}$ (**B**) -3 (**C**) 3 (**D**) 12 (**E**) None of these

2. In parallelogram $ABCD$, $m\angle C = 45°$. What is the measure of $\angle D$?

   (**A**) $45°$ (**B**) $90°$ (**C**) $120°$ (**D**) $\boxed{135°}$ (**E**) None of these

3. The measures of the angles of a triangle are in the ratio of $2 : 4 : 6$. Which of the following is the measure of one of the angles?

   (**A**) $15°$ (**B**) $45°$ (**C**) $\boxed{60°}$ (**D**) $120°$ (**E**) None of these

4. What is the distance between the points $(7, -2)$ and $(2, 3)$?

   (**A**) 5 (**B**) $\boxed{5\sqrt{2}}$ (**C**) 10 (**D**) $10\sqrt{2}$ (**E**) None of these

5. A regular polygon has interior angles that each measure $165°$. How many sides does this polygon have?

   (**A**) 16 (**B**) 20 (**C**) $\boxed{24}$ (**D**) 28 (**E**) None of these

6. In the diagram, O is the center of the circle, points A, B, C, and D all lie on the circle, and O is on segment $\overline{CD}$. If $m\angle BAC = 35°$, find $m\angle BCD$.

   (**A**) $35°$ (**B**) $40°$ (**C**) $\boxed{55°}$ (**D**) $60°$ (**E**) None of these

7. In right triangle ABC, $m\angle C = 30°$, $m\angle B = 90°$, and D is a point on $\overline{BC}$ such that $\overrightarrow{AD}$ is the angle bisector of $\angle BAC$. If $AB = 6$, find CD.

   (**A**) $\boxed{4\sqrt{3}}$ (**B**) $6\sqrt{3}$ (**C**) 10 (**D**) 12 (**E**) None of these

8. Two complementary angles have measures of $6x - 20$ and $4x$, both in degrees. Find the supplement of the smaller angle.

   (**A**) $130°$ (**B**) $134°$ (**C**) $\boxed{136°}$ (**D**) $169°$ (**E**) None of these

9. In the given figure, points C, D lie on line L_1, points A, B lie on line L_2, and lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect at point E. If L_1 is parallel to L_2, and $\overline{AB} \cong \overline{CD}$, then what else must be true?

   (**A**) $\overline{AD} \perp \overline{BC}$ (**B**) $\overline{BC} \cong \overline{AD}$ (**C**) $\overline{BE} \cong \overline{DE}$

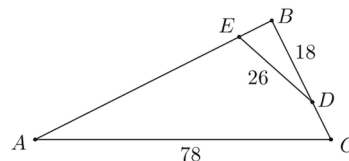   (**D**) $\boxed{\overline{AC} \cong \overline{BD}}$ (**E**) None of these

10. If the radius of a circle is doubled, what is the corresponding percent increase in the area?

    (**A**) 100% (**B**) 200% (**C**) $\boxed{300\%}$ (**D**) 400% (**E**) None of these

11. If the diagonals of a rhombus are 14 and 22 units long, find the area of the rhombus.

    (**A**) $36\ u^2$ (**B**) $77\ u^2$ (**C**) $\boxed{154\ u^2}$ (**D**) $308\ u^2$ (**E**) None of these

12. If the line containing points $(2,1)$ and $(-8,c)$ is parallel to the line containing points $(7, c+1)$ and $(11,1)$, what is the value of c?

(**A**) $-\dfrac{4}{5}$ (**B**) $\dfrac{1}{4}$ (**C**) $\dfrac{2}{7}$ (**D**) $\dfrac{8}{3}$ (**E**) $\boxed{\text{None of these}}$

13. In triangle $\triangle ABC$ shown, point E is on $\overline{AB}$, point D is on $\overline{BC}$, $AC = 78$, $BD = 18$, and $DE = 26$. If $CD = BE$ and $\angle BDE \cong \angle BAC$, find the perimeter of $\triangle ABC$.

(**A**) $\boxed{159}$ (**B**) 183 (**C**) 224 (**D**) 240 (**E**) None of these

14. Given a circle centered at $(3,4)$ that passes through point $(7,1)$, which of the following is the equation of the line tangent to the circle at point $(7,1)$?

(**A**) $3x - 4y = 17$ (**B**) $3x + 4y = 25$ (**C**) $4x - 3y = 17$ (**D**) $\boxed{4x - 3y = 25}$ (**E**) None of these

15. The perimeter of a rectangle is 56 units. The ratio of the length to the width is 4:3. The length, in units, of a diagonal of the rectangle is

(**A**) $\boxed{20}$ (**B**) 22 (**C**) 24 (**D**) 26 (**E**) None of these
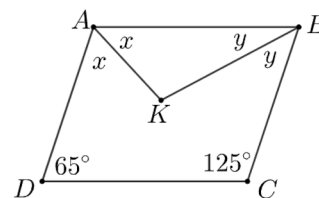
16. In triangle $\triangle ABC$, the midpoint of side $\overline{AB}$ is $(\frac{1}{2}, 3)$, the midpoint of $\overline{AC}$ is $(3, -3)$, and the midpoint of side $\overline{BC}$ is $(\frac{9}{2}, -1)$. Find the perimeter of $\triangle ABC$.

(**A**) $9 + 2\sqrt{2}$ (**B**) $9 + 4\sqrt{2}$ (**C**) $18 + 4\sqrt{2}$ (**D**) $\boxed{18 + 8\sqrt{2}}$ (**E**) None of these

17. Find the ratio of the volume of a cone of height 50m and radius 12m to the volume of a sphere of radius 30m.

(**A**) $\boxed{\dfrac{1}{15}}$ (**B**) $\dfrac{2}{3}$ (**C**) $\dfrac{5}{3}$ (**D**) $\dfrac{9}{1}$ (**E**) None of these

18. In the figure shown, two consecutive angles of a quadrilateral measure $65°$ and $125°$. The measure of angle $\angle DAK$ equals the measure of $\angle BAK$, and the measure of $\angle ABK$ equals the measure of $\angle CBK$. Find the degree measure of $\angle AKB$.

(**A**) $65°$ (**B**) $90°$ (**C**) $\boxed{95°}$ (**D**) $125°$ (**E**) None of these

19. When the point $(3,5)$ is reflected about the line $y = 4x - 2$, the resulting point is (a,b). Find $a + b$.

(**A**) $\dfrac{75}{11}$ (**B**) $\boxed{\dfrac{106}{17}}$ (**C**) $\dfrac{124}{23}$ (**D**) 8 (**E**) None of these

20. Let A be the measure of an acute angle whose complement measures one-fourth of its supplement. Let B equal the geometric mean of 4 and 36. Find the arithmetic mean of A and B.

(**A**) 24 (**B**) $\boxed{36}$ (**C**) 42 (**D**) 60 (**E**) None of these

21. An octahedron is a polyhedron with eight faces and six vertices. How many edges does it have?

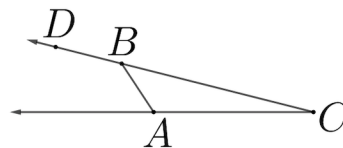(**A**) 8 (**B**) $\boxed{12}$ (**C**) 14 (**D**) 20 (**E**) None of these

22. If ray $\overrightarrow{OB}$ bisects angle $\angle AOC$ with $m\angle AOB = 2x + 10$ and $m\angle BOC = 8x - 14$ (each in degrees), then find $m\angle AOC$ in degrees.

(A) 24   (B) 28   (C) 32   (D) $\boxed{36}$ (E) None of these

23. Let $\triangle ABC$ be a triangle such that $AB = 4$, $AC = 10$ and $BC = 12$. Let T be a point on $\overrightarrow{CA}$ such that $\overrightarrow{BT}$ bisects the exterior angle $\angle ABD$. Find AT.

(A) 2   (B) 3   (C) 4   (D) $\boxed{5}$ (E) None of these

24. An 18 inch diameter pizza is cut into 12 slices. What is the distance around one slice?

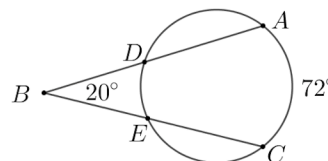(A) $\boxed{18 + \dfrac{3\pi}{2}}$ (B) $18 + 3\pi$ (C) $36 + \dfrac{3\pi}{2}$ (D) $36 + 3\pi$ (E) None of these

25. What is the sum of the interior and exterior angles of a convex, irregular decagon?

(A) $\boxed{1800°}$ (B) $1980°$ (C) $2160°$ (D) $2340°$ (E) Not enough information

26. In the figure provided, $\overline{AB}$ intersects the circle at D and A, $\overline{BC}$ intersects the circle at C and E, $m\angle ABC = 20°$, and $m\overset{\frown}{AC} = 72°$. What is the measure of the minor arc $\overset{\frown}{DE}$, in degrees?

(A) 26   (B) $\boxed{32}$ (C) 52   (D) 64   (E) None of these

27. An isosceles triangle has two sides of length 10 and one side of length 12. What is the area of the triangle?

(A) $\boxed{48}$ (B) 60   (C) 96   (D) 120   (E) None of these

28. The side of a square is the same length as the altitude of an equilateral triangle. If the area of the square is k times the area of the triangle, find k.
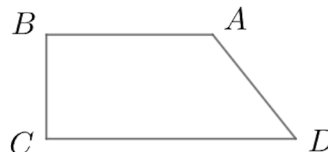
(A) $\sqrt{2}$ (B) $3\sqrt{2}$ (C) $\boxed{\sqrt{3}}$ (D) $2\sqrt{3}$ (E) None of these

29. The angles of a pentagon are in arithmetic progression with a common difference between each angle of $4°$. Find the measure of the largest angle.

(A) $108°$ (B) $\boxed{116°}$ (C) $120°$ (D) $124°$ (E) None of these

30. In trapezoid $ABCD$ shown, $AB = BC$, $AC = 15\sqrt{2}$, $AD = 17$, and $m\angle ABC = 90°$. Find the area of the trapezoid.
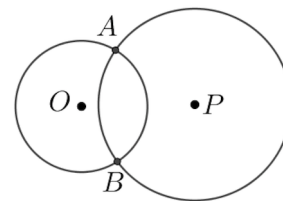
(A) $\boxed{285 \ u^2}$ (B) $345 \ u^2$ (C) $510 \ u^2$

(D) $570 \ u^2$ (E) None of these

31. A chord of length 18 is drawn in a circle of radius 11. Find the distance from the chord to the center of the circle.
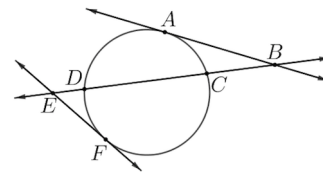
(A) 2   (B) 7   (C) $\sqrt{203}$ (D) $\boxed{2\sqrt{10}}$ (E) None of these

32. The circle with center O intersects the circle with center P at points A and B, as shown in the figure. If the measure of $\widehat{AB}$ on circle P is $50°$, and the measure of $\widehat{AB}$ on circle O is $80°$, what is the measure of angle $\angle OBP$?
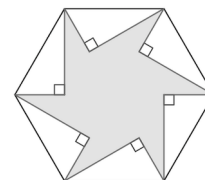
(**A**) $90°$ (**B**) $\boxed{115°}$ (**C**) $130°$ (**D**) $147.5°$ (**E**) None of these

33. If the length of the diagonal of a square is $x + y$, then what is the area of the square, in terms of x and y?

(**A**) $x^2 + y^2$ (**B**) $\dfrac{x^2 + y^2}{2}$ (**C**) $x^2 + 2xy + y^2$ (**D**) $\boxed{\dfrac{x^2 + 2xy + y^2}{2}}$ (**E**) None of these

34. A cake in the shape of a right circular cylinder with diameter of 8 inches is cut into six slices of equal volume. A second cake in the shape of a cube is cut into four slices of equal volume. The volume of a slice of the cylindrical cake is equal to the volume a slice of the cubical cake, and the cylindrical cake is twice the height of the cubical cake. Find area of the five faces on the cubical cake that need to be covered with icing.

(**A**) $\boxed{\dfrac{320\pi}{3}}$ (**B**) $\dfrac{512\pi\sqrt{3\pi}}{9}$ (**C**) $16\pi\left(1 + \dfrac{8\pi\sqrt{3\pi}}{3}\right)$ (**D**) $\dfrac{1280\pi}{3}$ (**E**) None of these

35. A triangle with vertices A, B, and C has the measure of $\angle BAC$ as twice the measure of $\angle ABC$. The measure of angle $\angle ABC$ is three times the measure of $\angle ACB$. Find the measure of $\angle ABC$, in degrees.

(**A**) 18 (**B**) 20 (**C**) $\boxed{54}$ (**D**) 60 (**E**) None of these

36. Right triangle $\triangle ABC$ is similar to right triangle $\triangle DEF$, which has sides in the ratio of 3:4:5. If the hypotenuse of $\triangle ABC$ is $\sqrt{5}$, find the length of the shorter leg of $\triangle ABC$.

(**A**) $\sqrt{3}$ (**B**) $\sqrt{5}$ (**C**) $\dfrac{\sqrt{5}}{5}$ (**D**) $\boxed{\dfrac{3\sqrt{5}}{5}}$ (**E**) None of these

37. The length of a rectangle is reduced by 10 feet, while the width of the same rectangle is increased by 6 feet. If the resulting figure is a square whose area is equal to that of the original rectangle, what is the perimeter of the original rectangle?

(**A**) 32 feet (**B**) $\boxed{68 \text{ feet}}$ (**C**) 128 feet (**D**) 225 feet (**E**) None of these

38. If 8 times the reciprocal of the circumference of a circle equals the diameter of the circle, then what is the area of the circle?

(**A**) $\boxed{2}$ (**B**) 4 (**C**) 4π (**D**) 16π (**E**) None of these

39. A circle is inscribed in a triangle with side lengths 7, 9, and 10. Find the area of the inscribed circle.

(**A**) $\dfrac{64\pi}{13}$ (**B**) $\boxed{\dfrac{72\pi}{13}}$ (**C**) $\dfrac{49\pi}{26}$ (**D**) $\dfrac{75\pi}{26}$ (**E**) None of these

40. The minute hand of a town clock measures 12 feet. How far, rounded to the nearest foot, does the tip of the hand travel in 35 minutes?

(**A**) 19 feet (**B**) 22 feet (**C**) 38 feet (**D**) $\boxed{44 \text{ feet}}$ (**E**) None of these

41. Line $\overleftrightarrow{AB}$ is tangent to the circle at point A, line $\overleftrightarrow{BE}$ intersects the circle at points C and D, and line $\overleftrightarrow{EF}$ is tangent to the circle at point F. If $AB = 6$, $BC = 4.5$, and $BE = 10$, find EF.

(**A**) 2   (**B**) 6   (**C**) $\sqrt{7}$ (**D**) $\boxed{\sqrt{11}}$ (**E**) None of these

42. Find the area of a triangle having side lengths 10, 20, and 26.

(**A**) 130   (**B**) 260   (**C**) $\boxed{24\sqrt{14}}$ (**D**) $24\sqrt{4830}$ (**E**) None of these

43. In $\triangle ABC$, we have that $AB = 9$, $AC = 9$ and $BC = 2$. If D is a point on $\overrightarrow{BC}$ such that C is between B and D, and $AD = 12$, find CD.

(**A**) $\boxed{7}$ (**B**) 8   (**C**) 9   (**D**) 10   (**E**) None of these

44. In a regular polygon, a total of 9 diagonals can be drawn. What is the measure of an interior angle of this polygon?

(**A**) 108°   (**B**) $\boxed{120°}$ (**C**) 135°   (**D**) 140°   (**E**) None of these

45. In rectangle $ABCD$, points F and G lie on $\overline{AB}$ such that $AF = FG = GB$, and E is the midpoint of $\overline{CD}$. Segment $\overline{AC}$ intersects segments $\overline{EF}$ and $\overline{EG}$ at points H and J, respectively. If the area of rectangle $ABCD$ is 70, find the area of $\triangle EHJ$.

(**A**) $\boxed{3}$ (**B**) 9   (**C**) $\dfrac{35}{9}$ (**D**) $\dfrac{70}{9}$ (**E**) None of these

46. Two points P and Q lie on line segment $\overline{AB}$, such that both P and Q are on the same side of the midpoint of $\overline{AB}$ as point A. Point P divides AB in the ratio of 2:3, and Q divides AB in the ratio of $3 : 4$. If $PQ = 4$, what is AB?

(**A**) 8   (**B**) 35   (**C**) 48   (**D**) $\boxed{140}$ (**E**) None of these

47. A saw blade is made by cutting six triangles, each with angle measures 30°, 60° and 90°, out of a regular hexagon. If the length of the longest diagonal of the hexagon is 12 inches, what is the area of the saw blade?

(**A**) $18\sqrt{3}$ (**B**) $\boxed{27\sqrt{3}}$ (**C**) $36\sqrt{3}$ (**D**) $54\sqrt{3}$ (**E**) None of these

48. For an angle $0 \leq \theta \leq \frac{\pi}{2}$, if $\sin\theta = 3\cos\theta$, what is $\tan\theta$?

(**A**) $\dfrac{1}{8}$ (**B**) $\dfrac{1}{3}$ (**C**) $\boxed{3}$ (**D**) 8   (**E**) None of these

49. Suppose a right square pyramid has a square base with side length 4 and a surface area of 40. What is the height of this pyramid?

(**A**) 2   (**B**) 3   (**C**) $\boxed{\sqrt{5}}$ (**D**) $\sqrt{6}$ (**E**) None of these

50. Suppose $\triangle ABC$ is an acute triangle with $AB = \sqrt{37}$, and $BC = 4\sqrt{3}$. Which of the following could be the length of the side $\overline{AC}$?

(**A**) $\boxed{9}$ (**B**) 10   (**C**) 11   (**D**) 12   (**E**) None of these