Alabama Statewide Math Contest - Round 2 Division Two

University of North Alabama

April 9, 2022

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Round 2

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

Let $\triangle ABC$ and $\triangle DEF$ be two triangles, with $\angle C$ and $\angle F$ both right angles, $\angle A \cong \angle E$ and $\angle B \cong \angle D$. If $AC = 3$, $DF = 4$, and $EF = 6$, find AB.

Geometry Question # 1

Answer:

Geometry Question # 1

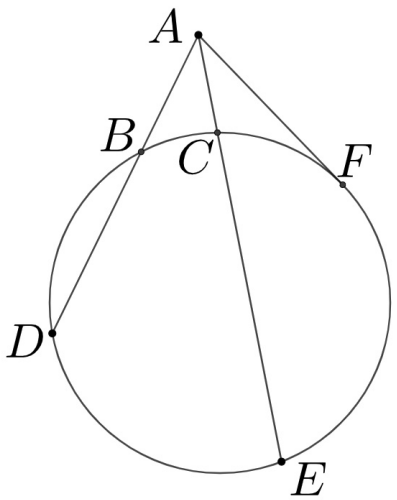
Answer: $\sqrt{13}$

Geometry Question # 2

Geometry Question # 2

RESET

Let points B, C, D, E, F be on the circle as shown, and A be an external point such that B is on $\overline{AD}$, C is on $\overline{AE}$, and $\overline{AF}$ is tangent to the circle at F. If $AB = 3$, $CE = 10$, and $AD = AE - 4$, find AF.



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $2\sqrt{6}$

Round 2

Algebra II

Algebra II Question # 3

Algebra II Question # 3

RESET

Find all solutions to the equation $|x+2| = |x^2-4|$.

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: $-2, 1, 3$

Algebra II Question # 4

Algebra II Question # 4

RESET

The sum of two numbers is 100. Their difference is 19. Find the smaller number.

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: $\frac{81}{2}$

Round 2

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET		

If $\sin x = \frac{3}{5}$, where $0 \leq x \leq \frac{\pi}{2}$, what is $\sin(2x)$?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: $\frac{24}{25}$

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Find all solutions to $(x+2)^2 = (2x-3)^2$.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{1}{3}$ and 5

Round 2

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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If $4^x = 8^{2x-1}$, find x.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: $\frac{3}{4}$

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

How many distinct arrangements are there of the following shapes?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 20

Round 2

Team

Team Question # 9

Team Question # 9

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The equation $x^3 - 12x^2 + 44x - 48 = 0$ has its three roots in arithmetic progression. What is the median of the three roots?

Team Question # 9

Answer:

Team Question # 9

Answer: 4

Team Question # 10

Team Question # 10

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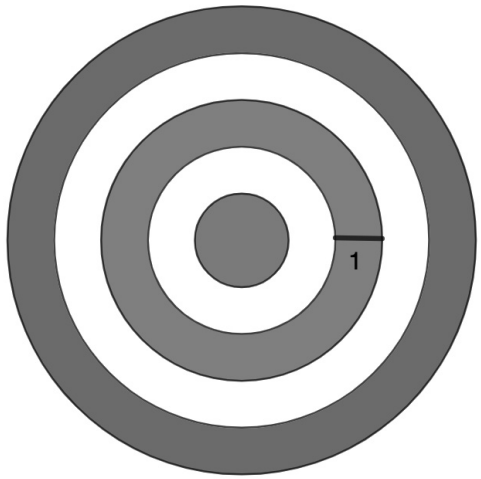
A circle of radius 1 is surrounded by concentric circles, with each ring (both shaded and unshaded) being 1 unit in width, as shown in the figure. Find the total area of the shaded regions.



Team Question # 10

Answer:

Team Question # 10

Answer: 15π

End of Round 2