Alabama Statewide Math Contest - Round 4 Division Two

University of North Alabama

April 9, 2022

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Round 4

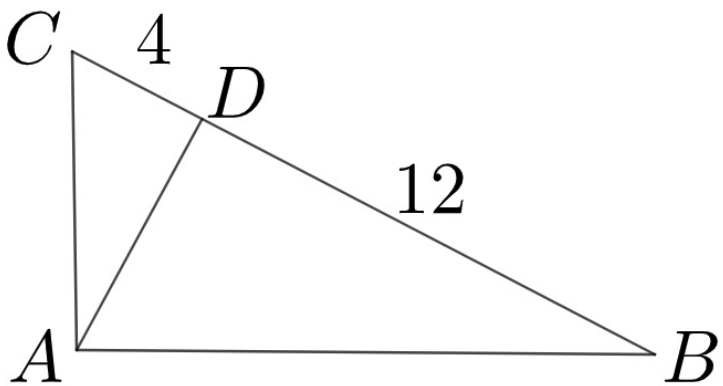
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

Given $\triangle ABC$ with $m\angle BAC = 90°$, $\overline{AD} \perp \overline{BC}$, $BD = 12$, and $CD = 4$, find AD.



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: $4\sqrt{3}$

Geometry Question # 2

Geometry Question # 2

RESET

Let A, B, C be points on a circle such that BC is a diameter, and $AB = AC = 4$. Find the area of the circle.

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 8π

Round 4

Algebra II

Algebra II Question # 3

Algebra II Question # 3

RESET

Given that 1 is a root of $f(x) = 2x^3 + x^2 - 5x + 2$, find the positive difference of the other two roots.

Algebra II Question # 3

Answer:

Algebra II Question # 3

Answer: $\frac{5}{2}$

Algebra II Question # 4

Algebra II Question # 4

RESET

How many vertical asymptotes does the graph of

$$f(x) = \frac{4 - x}{2x^2 - 7x - 4}$$

have?

Algebra II Question # 4

Answer:

Algebra II Question # 4

Answer: 1

Round 4

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET		

A box contains 80 blue, 24 white, and an unknown number of red marbles. If the probability of drawing a red marble is 0.2, how many red marbles are in the box?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 26

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Find the distance between $2+3\sqrt{2}i$ and $-1+\sqrt{2}i$ in the complex plane.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\sqrt{17}$

Round 4

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

Find x_{50}, the 50th term of the arithmetic sequence that begins

$$x_1 = 5, x_2 = 9, x_3 = 13, ...$$

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 201

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

Find the minimum value of the function $f(x) = 8x^2 + 4x + 5$.

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{9}{2}$

Round 4

Team

Team Question # 9

Team Question # 9

RESET		

For how many real number values on the interval $[-3, 0]$ is the function $f(x) = 2\tan x + \dfrac{x+5}{x^2-7}$ undefined?

Team Question # 9

Answer:

Team Question # 9

Answer: 2

Team Question # 10

Team Question # 10

RESET

A right square pyramid with a base of side length 8 has a height of 6. What is the surface area of this pyramid?

Team Question # 10

Answer:

Team Question # 10

Answer: $64 + 32\sqrt{13}$

End of Round 4