The 38th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 23, 2019 at Regional Testing Centers
Second Round: April 6, 2019 at The University of Alabama at Birmingham

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.
- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Editing by Miranda Bowie and Ashley Johnson, The University of North Alabama

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

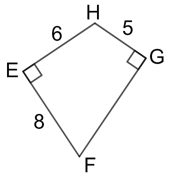
- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. A rectangle with sides of length 6 and 10 is rolled to make a cylinder of maximum height. One entire edge is joined with its opposite edge. A circular disk is placed at each end of the cylinder. Find the surface area of the resulting cylinder.

(**A**) $60 + \frac{9}{\pi}$ (**B**) $\boxed{60 + \frac{18}{\pi}}$ (**C**) 69 (**D**) 78 (**E**) None of these

2. In the figure shown, $\square EFGH$ is a quadrilateral with right angles at vertices E and G, $EF = 8$, $EH = 6$, and $HG = 5$. Find GF.



(**A**) 3 (**B**) 4 (**C**) $\boxed{5\sqrt{3}}$ (**D**) $5\sqrt{5}$ (**E**) None of these

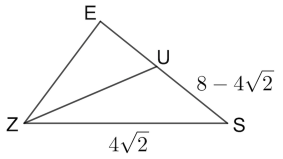
3. The minute hand of a school clock measures 6 inches. How far, measured in inches, does the tip of the hand travel in 25 minutes?

(**A**) 2.5 (**B**) $\boxed{5\pi}$ (**C**) 15π (**D**) 150 (**E**) None of these

4. What is the circumference of a circle with an area of 125π units squared?

(**A**) 5π (**B**) $5\sqrt{5}\,\pi$ (**C**) 10π (**D**) $\boxed{10\sqrt{5}\,\pi}$ (**E**) None of these

5. Triangle ZES shown is isosceles with $ZE = ES$, point U is on $\overline{ES}$ such that $\overrightarrow{ZU}$ bisects angle $\angle EZS$. If $ZS = 4\sqrt{2}$, and $US = 8 - 4\sqrt{2}$, find the length of $\overline{EU}$.



(**A**) $\boxed{4\sqrt{2} - 4}$ (**B**) 4 (**C**) $8 - 4\sqrt{2}$ (**D**) 8 (**E**) None of these

6. In $\triangle ABC$, the exterior angle at A has measure 95° and the exterior angle at C has measure 160°. Find the measure of $\angle ABC$.

(**A**) 85° (**B**) $\boxed{75^\circ}$ (**C**) 65° (**D**) 55° (**E**) None of these

7. The sides of a right triangle are a, $a + d$, and $a + 2d$ with a and d positive numbers. Find the value $\frac{a}{d}$.

(**A**) $\boxed{3}$ (**B**) 4 (**C**) 5 (**D**) 6 (**E**) None of these

8. The measure of each interior angle of a regular polygon is twelve times that of the measure of each central angle of the polygon. Find the number of sides in the regular polygon.

(**A**) 20 (**B**) 22 (**C**) 24 (**D**) $\boxed{26}$ (**E**) None of these

9. In parallelogram $ABCD$, the measure of angle $\angle BCD$ is 56°. What is the measure of angle $\angle CDA$?

(**A**) 34° (**B**) 56° (**C**) 72° (**D**) 112° (**E**) $\boxed{\text{None of these}}$

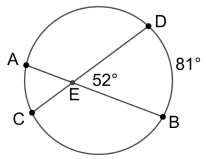
10. In square $ABCD$, point E is on side $\overline{DC}$ such that $CE = 2$ and $BE = 5$. What is the area of trapezoid $BEDA$?

(**A**) $\boxed{21 - \sqrt{21}}$ (**B**) $21 - 2\sqrt{21}$ (**C**) $29 - \sqrt{29}$ (**D**) $29 - 2\sqrt{29}$ (**E**) None of these

11. Find the area of the sector of a circle with radius 15 and arc of 48°.

(**A**) 4π (**B**) 8π (**C**) 15π (**D**) $\boxed{30\pi}$ (**E**) None of these

12. In the figure shown, $\overline{AB}$ and $\overline{CD}$ are chords of the circle with intersection point E. The measure of angle $\angle DEB$ is 52°, the measure of arc $\overset{\frown}{DB}$ is 81°, and the measure of arc $\overset{\frown}{AD}$ is equal to the measure of arc $\overset{\frown}{CB}$. Find the measure of arc $\overset{\frown}{AD}$.

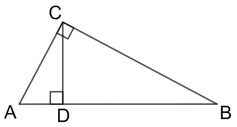


(**A**) 99° (**B**) 125° (**C**) $\boxed{128°}$ (**D**) 157° (**E**) None of these

13. The ratio of two angles is 5 to 7, and their sum is 72°. What is the measure of the larger angle minus the smaller angle?

(**A**) 8° (**B**) 10° (**C**) $\boxed{12°}$ (**D**) 14° (**E**) None of these

14. In the right triangle ABC shown, $\angle ACB$ is a right angle, D is on $\overline{AB}$, $\overline{CD} \perp \overline{AB}$, $AD = 4$, and $CD = 6$. Find the area of triangle ABC in square units.



(**A**) 36 (**B**) $\boxed{39}$ (**C**) 45 (**D**) 48 (**E**) None of these

15. If the volume of a sphere is $\frac{\pi}{6}$, then its surface area is:

(**A**) 4 (**B**) 12 (**C**) $\frac{\pi}{3}$ (**D**) $\boxed{\pi}$ (**E**) None of these

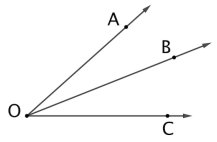
16. The sum of the measures of the complement and the supplement of an angle is 196°. What is the measure of the angle?

(**A**) 16° (**B**) $\boxed{37°}$ (**C**) 74° (**D**) 106° (**E**) None of these

17. Find the area of a regular hexagon with a radius of 8, in square units.

(**A**) $\boxed{96\sqrt{3}}$ (**B**) $72\sqrt{3}$ (**C**) 192 (**D**) 48 (**E**) None of these

18. In the figure given, ray $\overrightarrow{OB}$ bisects angle $\angle AOC$. Given the measure of angle $\angle AOB$ is $3x + 12$, and the measure of angle $\angle BOC$ is $18x - 18$, find the supplement of the measure of angle $\angle AOC$.



(**A**) 42° (**B**) 54° (**C**) 138° (**D**) $\boxed{144°}$ (**E**) None of these

19. The side lengths of a polygon are 2, 3, 4, 5, and 6. Find the perimeter of a similar polygon whose longest side is 15.

(**A**) 20 (**B**) 35 (**C**) $\boxed{50}$ (**D**) 65 (**E**) None of these

20. A cone has a height that is six times its radius. If the volume of the cone is 128π, what is the radius of the cone?

(**A**) $\boxed{4}$ (**B**) 8 (**C**) 16 (**D**) 24 (**E**) None of these

21. The consecutive angles of a pentagon form an arithmetic sequence. If the smallest angle measures 60 degrees, what is the measure of the largest angle?

(**A**) $96°$ (**B**) $\boxed{156°}$ (**C**) $180°$ (**D**) $300°$ (**E**) None of these

22. Two rectangles contain the same area, 480 square yards. The difference of their lengths is 10 yards, and of their widths is 4 yards. Find the sum of the perimeters of the two rectangles.

(**A**) 188 (**B**) $\boxed{196}$ (**C**) 200 (**D**) 244 (**E**) None of these

23. In right triangle $\triangle ABC$, $m\angle ACB = 90°$, $AC = 12$, and $AB = 3BC$. What is the length of the hypotenuse?

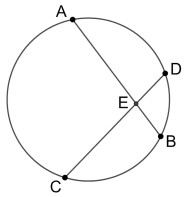
(**A**) 13 (**B**) 18 (**C**) $5\sqrt{2}$ (**D**) $\boxed{9\sqrt{2}}$ (**E**) None of these

24. What is the area of a triangle whose sides have lengths 8, 10, and 12?

(**A**) 30 (**B**) 40 (**C**) $\boxed{15\sqrt{7}}$ (**D**) $60\sqrt{33}$ (**E**) None of these

25. In the diagram shown, a circle has chords $\overline{CD}$, and $\overline{AB}$ that intersect at point E. Let $CE = 12$, $DE = 3$, $BE = x$, and $AE = 2x + 1$. How much longer is $\overline{AE}$ than $\overline{BE}$?

(**A**) $\frac{3}{2}$ (**B**) $\boxed{5}$ (**C**) $\frac{17}{3}$ (**D**) 9 (**E**) None of these



26. The area of a rhombus is 130 square units, and one diagonal has length 13 units. What is the length of the other diagonal?

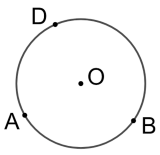
(**A**) 5 (**B**) 10 (**C**) $\boxed{20}$ (**D**) 30 (**E**) None of these

27. If all but one of the interior angles of a convex polygon add up to $860°$, what is the measure of the remaining angle?

(**A**) $\boxed{40°}$ (**B**) $60°$ (**C**) $80°$ (**D**) $100°$ (**E**) None of these

28. In the given circle, chord $\overline{AB}$ has length 12 and the measure of arc $\overset{\frown}{ADB}$ is $240°$. What is the radius of the circle?

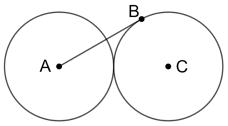
(**A**) 4 (**B**) 6 (**C**) $2\sqrt{3}$ (**D**) $\boxed{4\sqrt{3}}$ (**E**) None of these



29. If a square and a circle have the same perimeter, then the area of the square divided by the area of the circle is

(**A**) $\boxed{\frac{\pi}{4}}$ (**B**) $\frac{\pi}{2}$ (**C**) π (**D**) 2π (**E**) None of these

30. In the figure shown, the circle centered at A and the circle centered at C both have radius 6, and are externally tangent. Let B be a point on the circle centered at C so that $\overleftrightarrow{AB}$ is a tangent. Determine the length of the line segment $\overline{AB}$.

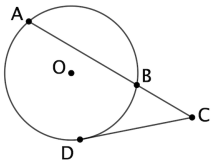


(**A**) 5 (**B**) 13 (**C**) $6\sqrt{2}$ (**D**) $\boxed{6\sqrt{3}}$ (**E**) None of these

31. The point $(5, 1)$ is rotated $90°$ clockwise about the origin. What is the distance between the new point and the original?

(**A**) $4\sqrt{2}$ (**B**) $6\sqrt{2}$ (**C**) $\boxed{2\sqrt{13}}$ (**D**) $4\sqrt{13}$ (**E**) None of these

32. In the figure, points A, B, and D lie on the circle centered at O, with $\overleftrightarrow{CD}$ tangent to the circle at point D. If $CD = 12$ and $BC = 8$, find the value of AB.

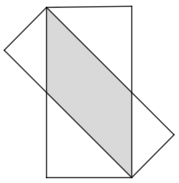


(**A**) 4 (**B**) $\boxed{10}$ (**C**) 12 (**D**) 18 (**E**) None of these

33. A rectangular piece of paper is folded in half five times in succession. The final folded piece of paper has an area of 15 units squared. Find the area of the original rectangular piece of paper.

(**A**) 120 (**B**) 240 (**C**) $\boxed{480}$ (**D**) 960 (**E**) None of these

34. Two congruent rectangles each measuring 6 inches by 14 inches overlap as shown. What is the area of the shaded region in square inches?

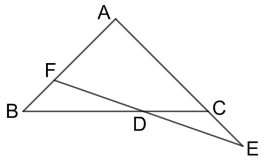


(**A**) $\frac{240}{7}$ (**B**) 42 (**C**) 48 (**D**) $\boxed{\frac{348}{7}}$ (**E**) None of these

35. A line parallel to $2x + 5y - 8 = 0$ contains points $(3, 7)$ and $(k, 3)$. Find k.

(**A**) -7 (**B**) $\frac{7}{5}$ (**C**) $\frac{23}{5}$ (**D**) $\boxed{13}$ (**E**) None of these

36. Let $\triangle ABC$ be an isosceles triangle with $AC = AB$, and let D be a point on $\overline{CB}$. Extend $\overline{AC}$ to a point E so that $CD = CE$. Extend $\overline{ED}$ to meet $\overline{AB}$ at point F. If angle $\angle CED$ measures 10 degrees, find the measure of angle $\angle AFE$ in degrees.



(**A**) $\boxed{30}$ (**B**) 50 (**C**) 60 (**D**) 80 (**E**) None of these

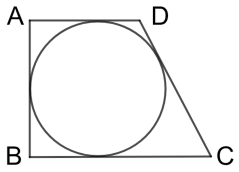
37. A cylindrical can with a diameter of 2 inches and a height of 4 inches contains six spherical meatballs, each 1 inch in diameter. If the rest of the can is full of gravy, what is the volume of the gravy?

(**A**) π (**B**) 2π (**C**) $\boxed{3\pi}$ (**D**) 4π (**E**) None of these

38. If the measure of $\angle B$ is four times the measure of $\angle A$, and the measure of the complement of $\angle A$ is four times the measure of the complement of $\angle B$, what is the measure of the supplement of $\angle A$?

(**A**) $36°$ (**B**) $54°$ (**C**) $116°$ (**D**) $\boxed{162°}$ (**E**) None of these

39. In trapezoid $ABCD$, where $m\angle DAB = 90°$, a circle is inscribed as shown. The circle's point of tangency with $\overline{CD}$ divides $\overline{CD}$ into lengths of 4 and 16. Find the area of the trapezoid in units squared.



(**A**) 216 (**B**) 264 (**C**) $\boxed{288}$ (**D**) 352 (**E**) None of these

40. A regular polygon with k sides has that each interior angle measures 135°. Find k.

(**A**) $\boxed{8}$ (**B**) 9 (**C**) 10 (**D**) 11 (**E**) None of these

41. Triangle $\triangle ABC$ has side lengths 5, 6, and 7 with an area of $6\sqrt{6}$. Find the sum of the three altitudes of the triangle.

(**A**) $\boxed{\frac{214\sqrt{6}}{35}}$ (**B**) $\frac{35\sqrt{6}}{2}$ (**C**) 18 (**D**) 36 (**E**) None of these

42. Find the center of the circle given by the equation $8x^2 + 8y^2 - 32x - 128y = 105$.

(**A**) $(-2, -8)$ (**B**) $\boxed{(2, 8)}$ (**C**) $(-8, -2)$ (**D**) $(8, 2)$ (**E**) None of these

43. For a given angle A in the first quadrant, $\tan A = \frac{5}{6}$. Find $\cos A$.

(**A**) $\frac{5\sqrt{61}}{61}$ (**B**) $\boxed{\frac{6\sqrt{61}}{61}}$ (**C**) $\frac{\sqrt{11}}{6}$ (**D**) $\frac{6\sqrt{11}}{11}$ (**E**) None of these

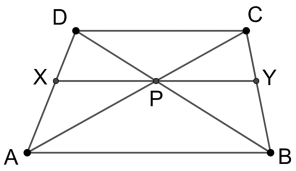
44. In parallelogram $PQRS$, vertices P, Q, and R have coordinates $(-3, -2)$, $(1, 5)$ and $(9, 1)$, respectively. Find the coordinates of vertex S.

(**A**) $(4, -5)$ (**B**) $(5, 4)$ (**C**) $\boxed{(5, -6)}$ (**D**) $(6, -5)$ (**E**) None of these

45. Which of the following points lie on the circle of radius 3 centered at $(0, 2)$?

(**A**) $(3, 2\sqrt{2})$ (**B**) $(\sqrt{3}, 2)$ (**C**) $\boxed{(\sqrt{5}, 0)}$ (**D**) $(-\sqrt{2}, -1)$ (**E**) None of these

46. In trapezoid $ABCD$, $CD = 20$ and $AB = 30$. Point P is the intersection of diagonals $\overline{BD}$ and $\overline{AC}$. Let X be the point on $\overline{AD}$ and Y be the point on $\overline{BC}$ so that $\overline{XY}$ goes through P and is parallel to $\overline{AB}$ and $\overline{CD}$. Find the length of $\overline{XY}$.



(**A**) $\boxed{24}$ (**B**) 25 (**C**) $16\sqrt{2}$ (**D**) $20\sqrt{2}$ (**E**) None of these

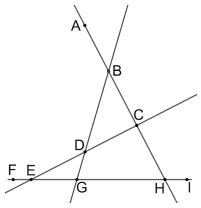
47. Find the perimeter of an equilateral triangle whose vertices lie on a circle with radius of 2 inches.

(**A**) $3\sqrt{2}$ (**B**) $3\sqrt{3}$ (**C**) $6\sqrt{2}$ (**D**) $\boxed{6\sqrt{3}}$ (**E**) None of these

48. A regular dodecahedron has twenty vertices. How many edges does it have?

(**A**) 12 (**B**) 20 (**C**) 24 (**D**) $\boxed{30}$ (**E**) None of these

49. In the figure, four straight lines intersect, and points A, B, C, D, E, F, G, H, and I are placed as shown. What is the value of $m\angle ABD + m\angle FED + m\angle GDC + m\angle CHI$?



(**A**) $360°$ (**B**) $450°$ (**C**) $480°$ (**D**) $\boxed{540°}$ (**E**) None of these

50. Find the height of a right pyramid with a square base whose area is 16 square units and whose lateral edges have length 5 units.

(**A**) 5 (**B**) $\boxed{\sqrt{17}}$ (**C**) $\sqrt{21}$ (**D**) $\sqrt{29}$ (**E**) None of these