GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overarc{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\overarc{AB}$ is the measure of $\overarc{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. The measures of the exterior angles of a hexagon are x, $2x$, $3x$, $3x$, $4x$, and $5x$. Find the measure of the largest interior angle.
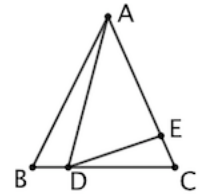
   (A) $20°$ **(B)** $100°$ **(C)** $120°$ **(D)** $\boxed{160°}$ **(E)** None of these

2. A right circular cylinder has a diameter of 6 and a lateral area of 60π. Find the volume of the cylinder in cubic units.

   (A) 60π **(B)** $\boxed{90\pi}$ **(C)** 120π **(D)** 160π **(E)** None of these

3. In the triangle shown, the measure of $\angle BAD = 38°$, $AB = AC$, and $AD = AE$. Find the measure of $\angle CDE$.

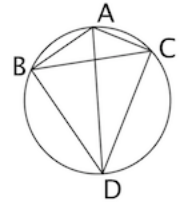   **(A)** $\boxed{19°}$ **(B)** $27°$ **(C)** $38°$ **(D)** $52°$ **(E)** None of these

4. The perimeter of a rectangle is 56 inches. The ratio of its length to its width is 4:3. The length, in inches, of a diagonal of the rectangle is

   (A) 15 **(B)** $\boxed{20}$ **(C)** 35 **(D)** 40 **(E)** None of these

5. Let A, B, and C be points on a circle of radius 3. In $\triangle ABC$, $\angle ACB = 30°$ and $AC = 2$. Find the perimeter of $\triangle ABD$, if $\overline{AD}$ is the diameter of the circle.

   (A) $6 + 2\sqrt{3}$ **(B)** $8 + 2\sqrt{3}$ **(C)** $\boxed{9 + 3\sqrt{3}}$

   (D) $12 + 2\sqrt{3}$ **(E)** None of these

6. Find the area in square units of the sector of a circle determined by the central angle of $45°$ with a radius of 10 units.

   (A) 5π **(B)** 15π **(C)** 25π **(D)** 50π **(E)** $\boxed{\text{None of these}}$

7. The circumference of a circle is 100 inches. The side of a square inscribed in this circle, in inches, is

   (A) $25\sqrt{2}\pi$ **(B)** $\dfrac{25\sqrt{2}}{\pi}$ **(C)** $50\sqrt{2}\pi$ **(D)** $\boxed{\dfrac{50\sqrt{2}}{\pi}}$ **(E)** None of these

8. In trapezoid $ABCD$, $\angle ABC$ is a right angle, and the diagonal $\overline{BD}$ is perpendicular to the leg $\overline{AD}$. The length of leg $\overline{BC}$ is 3, and the length of the diagonal $\overline{BD}$ is 5. Find the area of trapezoid $ABCD$.

   (A) 15 **(B)** 30 **(C)** $\boxed{\dfrac{123}{8}}$ **(D)** $\dfrac{1107}{32}$ **(E)** None of these
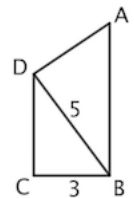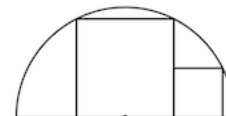
9. The lengths of two sides of a triangle are 5 and 8. If x is the length of the third side, what are the possible values of x?

   (A) $3 < x < 8$ **(B)** $\boxed{3 < x < 13}$ **(C)** $5 < x < 8$ **(D)** $5 < x < 13$ **(E)** $8 < x < 13$

10. Two squares are inscribed in a semicircle as shown. If the area of the smaller square is 100, what is the radius of the semicircle?

    (**A**) $5\sqrt{5}$ (**B**) $5\sqrt{13}$ (**C**) $10\sqrt{2}$ (**D**) $\boxed{10\sqrt{5}}$ (**E**) None of these

11. What is the measure of each interior angle of a regular decagon?

   (**A**) $\boxed{144°}$ (**B**) $150°$ (**C**) $165°$ (**D**) $216°$ (**E**) None of these

12. Triangle $\triangle ABC$ has a point D on $\overline{BC}$ such that D is a midpoint of $\overline{BC}$ and ray $\overrightarrow{AD}$ is a bisector of $\angle BAC$. Let G be the point where all three medians intersect. If $GD = 3$ and $BG = 7$, find the area of $\triangle ABC$.

    (**A**) $12\sqrt{10}$ (**B**) $14\sqrt{10}$ (**C**) $\boxed{18\sqrt{10}}$ (**D**) $20\sqrt{10}$ (**E**) None of these

13. In the figure shown, $ABCD$ and $DCFE$ are rectangles, with $AB = 4$, $AC = 5$, and $BC = CF$. What is the perimeter of $\triangle ACF$?

    (**A**) $\boxed{8 + 2\sqrt{13}}$ (**B**) $8 + 5\sqrt{2}$ (**C**) $9 + 4\sqrt{5}$

    (**D**) $10 + 5\sqrt{2}$ (**E**) None of these
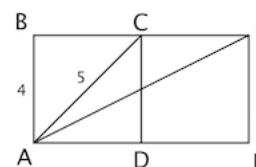
14. For a particular triangle $\triangle ABC$, the perpendicular bisectors of each side intersect at a point exterior to the triangle. What must be true about $\triangle ABC$?

    (**A**) It is scalene.   (**B**) It is isosceles.   (**C**) It is acute.   (**D**) It is right.   (**E**) $\boxed{\text{It is obtuse.}}$

15. Find the equation of the perpendicular bisector of the line segment whose endpoints are $(-2, 1)$ and $(3, -5)$.

    (**A**) $5y + 6x = -7$ (**B**) $5y - 6x = -13$ (**C**) $\boxed{12y - 10x = -29}$

    (**D**) $12y + 10x = -19$ (**E**) None of these

16. In the circle shown, point O is the center of the circle, and segment $\overline{OB}$ is 26 inches. Find the length of chord $\overline{AB}$, if chord $\overline{AB}$ is 24 inches from the center.

    (**A**) $6\sqrt{34}$ (**B**) $8\sqrt{21}$ (**C**) $\boxed{20}$ (**D**) 36 (**E**) None of these
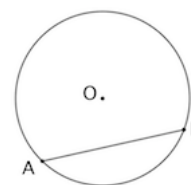
17. Two of the exterior angles of a pentagon have measure of $75°$ and $105°$. The measures of the remaining three exterior angles have the ratio 3:4:5. What is the median of the measures of the 5 exterior angles?

    (**A**) $65°$ (**B**) $72°$ (**C**) $\boxed{75°}$ (**D**) $120°$ (**E**) None of these

18. Let $ABCD$ be a quadrilateral, where diagonal $\overline{AC}$ is a perpendicular bisector of diagonal $\overline{BD}$ and $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$. Then $ABCD$ **must** be a:

    (**A**) Isosceles Trapezoid   (**B**) Rectangle   (**C**) $\boxed{\text{Rhombus}}$ (**D**) Right Trapezoid   (**E**) Square

19. Quadrilateral $MATH$ is inscribed in the circle with side lengths as shown. Find the area of the quadrilateral $MATH$, in square units.

(**A**) $\boxed{30}$ (**B**) 32 (**C**) $3\sqrt{21}$ (**D**) $6\sqrt{21}$ (**E**) None of these
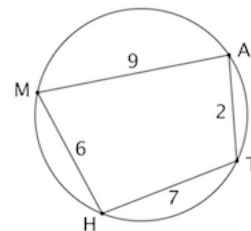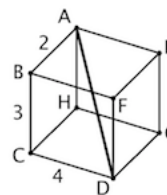
20. A cube has the same surface area as a rectangular solid whose dimensions are 16 inches long, 12 inches wide, and 9 inches high. Find the length of an edge of the cube, in inches.

(**A**) 12 (**B**) 24 (**C**) $\sqrt{39}$ (**D**) $\boxed{2\sqrt{37}}$ (**E**) None of these

21. A pole $\overline{AD}$ is to be placed in a rectangular box as shown, with dimensions of 2, 3, and 4 units. Find the length of the pole.

(**A**) $3\sqrt{3}$ (**B**) $\boxed{\sqrt{29}}$ (**C**) $\sqrt{34}$ (**D**) $\sqrt{41}$ (**E**) None of these

22. Given a right circular cylinder with radius of 5cm and height of 4cm, determine the total surface area, in square cm.

(**A**) 70π (**B**) 80π (**C**) $\boxed{90\pi}$ (**D**) 100π (**E**) None of these

23. Let $\triangle ABC$ be a scalene triangle whose area is 84 square units. Two side lengths are given by $AB = 10$ and $AC = 21$. Assuming side $\overline{AC}$ is opposite the largest angle, find the perimeter of the triangle.

(**A**) $\boxed{48}$ (**B**) 50 (**C**) 56 (**D**) 58 (**E**) None of these

24. In the figure shown, $ABCDEF$ is a regular hexagon, while $EFGHI$ is a regular pentagon. Find the measure of angle $\angle AGF$.

(**A**) $\boxed{24°}$ (**B**) $32°$ (**C**) $48°$ (**D**) $66°$ (**E**) None of these
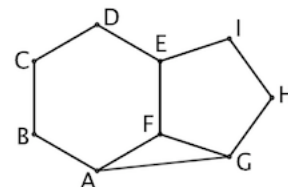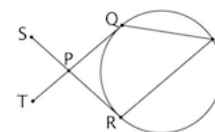
25. A square has perimeter $p > 0$ and area A. If $A = 2p$, what is the value of p?

(**A**) $\boxed{32}$ (**B**) 64 (**C**) $8\sqrt{2}$ (**D**) $16\sqrt{2}$ (**E**) None of these

26. If $\overline{TQ}$ and $\overline{SR}$ are tangents to the circle shown, with P on both $\overline{TQ}$ and $\overline{SR}$, and $m\angle SPT = 58°$, what is the measure of $\angle QLR$?

(**A**) $58°$ (**B**) $\boxed{61°}$ (**C**) $116°$ (**D**) $122°$ (**E**) None of these

27. What is the measure of an interior angle of an equiangular polygon with 54 diagonals?

(**A**) $108°$ (**B**) $120°$ (**C**) $135°$ (**D**) $\boxed{150°}$ (**E**) None of these

28. Let $\triangle ABC$ be a triangle with point D on $\overline{AB}$ such that $AC = CD = DB$ and $m\angle B = 23°$. Determine the measure of angle $\angle ACD$ in degrees.

(**A**) $60°$ (**B**) $67°$ (**C**) $69°$ (**D**) $\boxed{88°}$ (**E**) None of these

29. In the figure shown, two circles with radii 4 and 9 intersect in a single point. The straight line ℓ is tangent to both circles at points P and Q, respectively. Find the distance between P and Q.
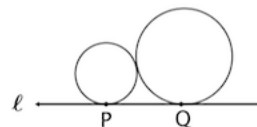
(**A**) 9   (**B**) 10   (**C**) $\boxed{12}$ (**D**) 13   (**E**) None of these

30. The sum of the measures of the complement and supplement of an angle is $196°$. Find the measure of the angle.

(**A**) $\boxed{37°}$ (**B**) $40°$ (**C**) $42°$ (**D**) $52°$ (**E**) None of these

31. Rectangle $ABCD$ can be divided into three congruent squares, as shown. If $ABCD$ has a perimeter of 168 inches, find the area of one of the squares, in square inches.

(**A**) 84   (**B**) 196   (**C**) 361   (**D**) $\boxed{441}$ (**E**) None of these

32. The area of a rhombus having side lengths of 37 inches is 840 square inches. Find the length of the longest diagonal.

(**A**) 28   (**B**) 35   (**C**) 56   (**D**) $\boxed{70}$ (**E**) None of these

33. A circle has points $(-2, -3)$ and $(6, 1)$ as endpoints of a diameter. What is the area of the circle?

(**A**) 10π (**B**) $\boxed{20\pi}$ (**C**) 40π (**D**) 80π (**E**) None of these

34. A circle of radius 2 is tangent to both sides of an angle. A circle of radius 3 is tangent to the circle of radius 2 and both sides of the same angle. A third circle is tangent to the circle of radius 3 and both sides of the same angle. Find the radius of the third circle.
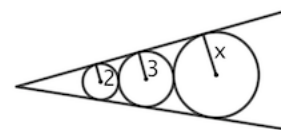
(**A**) 4   (**B**) $\boxed{4.5}$ (**C**) 5   (**D**) 5.5   (**E**) None of these

35. $LMNO$ is a square. P is a point inside the square such that LPO is an equilateral triangle. Determine the measure of $\angle PMN$.

(**A**) $\boxed{15°}$ (**B**) $30°$ (**C**) $60°$ (**D**) $75°$ (**E**) None of these

36. What is the surface area of a basketball inscribed in a cubical box (so that it touches all six faces) if the box has a surface area of 486 inches?

(**A**) 36π (**B**) $\boxed{81\pi}$ (**C**) 243π (**D**) 324π (**E**) None of these

37. In the trapezoid $PQRS$ shown, $\overline{PQ} \parallel \overline{RS}$, $PQ = 10$, $RS = 20$, $ST = 5$, and $QT = 12$. What is the length of $\overline{QW}$?
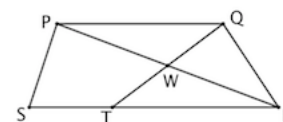
(**A**) 3.6   (**B**) $\boxed{4.8}$ (**C**) 6.0   (**D**) 8.2   (**E**) None of these

38. Find the diameter of the largest circle which can be inscribed in a triangle whose side lengths are 5, 12, and 13.

(**A**) 1   (**B**) 2   (**C**) 3   (**D**) $\boxed{4}$ (**E**) None of these

39. A square is surrounded by four congruent rectangles. If each of the rectangles has perimeter of 22 units, determine the total sum of the areas of the four rectangles and the interior square.

(**A**) 81   (**B**) $\boxed{121}$ (**C**) 225   (**D**) 484   (**E**) None of these
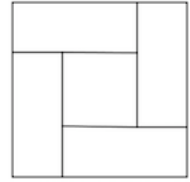
40. Find the radius of a circle if a central angle of 30° intercepts an arc of 11 inches. Round your answer to the nearest inch.

(**A**) 5   (**B**) 11   (**C**) $\boxed{21}$ (**D**) 33   (**E**) None of these

41. In the figure shown, points A, B, C, D and E all lie on the circle, point O is the center of the circle, and both $\overline{AD}$ and $\overline{CE}$ go through point O. Angle $\angle BEC$ has measure 28°, and $\angle ADB$ has measure 37°. Find the measure of $\angle ACE$.

(**A**) 14°   (**B**) 20°   (**C**) $\boxed{25°}$ (**D**) 37°   (**E**) None of these
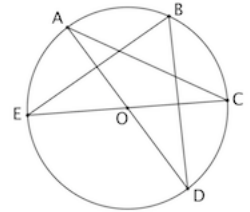
42. Let $\triangle ABC$ be a right triangle, with $m\angle ABC = 90°$, and $m\angle CAB = 30°$. Place a point D on $\overline{AB}$ 4 units from A such that $m\angle CDB = 60°$. What is the length of $\overline{BC}$?

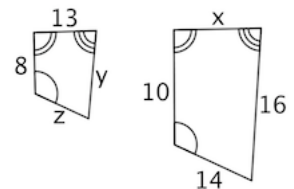(**A**) 2   (**B**) 6   (**C**) $\boxed{2\sqrt{3}}$ (**D**) $6\sqrt{3}$ (**E**) None of these

43. A right circular cone has for its base a circle having the same radius as a given sphere. The volume of the cone is one-half that of the sphere. What is the ratio of the height of the cone to the radius of its base?

(**A**) $\boxed{2:1}$ (**B**) $1:2$ (**C**) $2:3$ (**D**) $3:2$ (**E**) None of these

44. Two similar polygons are shown. Find the sum $x + y + z$.

(**A**) $\boxed{40.25}$ (**B**) 34.4   (**C**) 37   (**D**) 38   (**E**) None of these

45. Two lines that do not lie in the same plane and do not intersect are called:

(**A**) orthogonal   (**B**) non-Euclidean   (**C**) normal   (**D**) $\boxed{\text{skew}}$ (**E**) perpendicular
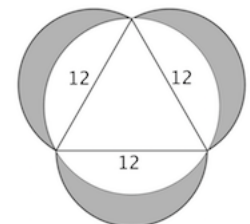
46. In square $ABCD$, point E is on side $\overline{CD}$ such that $CE = 2$ and $BE = 5$. What is the area of the square?

(**A**) 9   (**B**) 18   (**C**) $\boxed{21}$ (**D**) 29   (**E**) None of these

47. In the figure shown, the equilateral triangle with a side length 12 is inscribed in a circle, and the three outer arcs are semicircles. Determine the shaded area in square units.

(**A**) $16\pi - 12\sqrt{3}$ (**B**) $56\pi + 12\sqrt{3}$ (**C**) $12\pi + 24\sqrt{3}$

(**D**) $\boxed{6\pi + 36\sqrt{3}}$ (**E**) None of these

48. At 2:15, the hour and minute hands of a clock form an angle of:

(**A**) 20° (**B**) $\boxed{22.5°}$ (**C**) 27.5° (**D**) 30° (**E**) None of these

49. What are the points, if any, at which the circle $x^2 + (y-1)^2 = 1$ intersects the y-axis?

(**A**) $(2,0)$ and $(0,0)$ (**B**) $(1,1)$ (**C**) $\boxed{(0,2) \text{ and } (0,0)}$ (**D**) $(0,0)$ (**E**) None of these

50. A right square pyramid with a base area of 16 and a height of 6 is cut halfway up parallel to the base. What is the surface area of the bottom half?

(**A**) $12 + 12\sqrt{10}$ (**B**) $\boxed{20 + 12\sqrt{10}}$ (**C**) $12 + 24\sqrt{2}$ (**D**) $20 + 24\sqrt{2}$ (**E**) None of these