The 36th Annual

$\mathcal{ALABAMA}$

STATEWIDE MATHEMATICS CONTEST

First Round: February 25, 2017 at Regional Testing Centers
Second Round: April 8, 2017 at The University of North Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown and Luke A. Smith, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overparen{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\overparen{AB}$ is the measure of $\overparen{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.
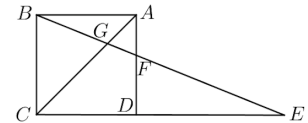
A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)
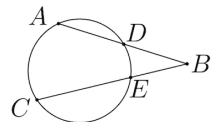
Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. Find the radius of a circle if a sector of area 30π has an arc of $48°$.

   (A) $\boxed{15}$ (B) $\sqrt{15}$ (C) 30 (D) $\sqrt{30}$ (E) None of these

2. Determine the surface area of a rectangular prism with length of 7 inches, width of 3 inches and height of 5 inches.

   (A) 71 in^2 (B) 105 in^2 (C) $\boxed{142\text{ in}^2}$ (D) 210 in^2 (E) None of these

3. A rectangle measuring 40 cm by 30 cm is inscribed in a circle. Find the circumference of the circle.

   (A) 35π (B) 40π (C) 55π (D) 75π (E) $\boxed{\text{None of these}=50\pi}$

4. $ABCD$ is a square as shown, with a line through B which intersects the extension of $\overline{CD}$ at point E, the side $\overline{AD}$ at point F and the diagonal $\overline{AC}$ at point G. If $BG = 3$ and $GF = 1$, find the length of $\overline{FE}$.
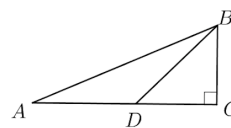
   (A) 4 (B) $4\sqrt{2}$ (C) $\dfrac{8\sqrt{2}}{5}$ (D) $\boxed{8}$ (E) None of these

5. An isosceles triangle with base 10 inches has an area of 60 square inches. Find the perimeter of the triangle, in inches.

   (A) $10 + 6\sqrt{5}$ (B) $10 + 10\sqrt{5}$ (C) 30 (D) $\boxed{36}$ (E) None of these

6. What is the shortest distance from the point $(-2, 3)$ to the circle given by $(x - 2)^2 + (y + 5)^2 = 5$?

   (A) 2 (B) 3 (C) $\boxed{3\sqrt{5}}$ (D) $4\sqrt{5}$ (E) None of these

7. The lengths of two corresponding sides of two similar polygons are 4 and 6. If the area of the smaller polygon is 20, find the area of the larger polygon.

   (A) 30 (B) 40 (C) $\boxed{45}$ (D) 60 (E) None of these

8. In the figure shown, D is on $\overline{AB}$, E is on $\overline{BC}$, $m\angle B = 15°$, $m\widehat{DE} = 50°$ and $m\widehat{AD} = m\widehat{CE}$. Determine $m\widehat{AD}$.

   (A) $65°$ (B) $80°$ (C) $\boxed{115°}$ (D) $122.5°$ (E) None of these

9. In parallelogram $ABCD$, $m\angle C = 65°$. Determine $m\angle D$.

   (A) $32.5°$ (B) $65°$ (C) $\boxed{115°}$ (D) $295°$ (E) None of these

10. The area of a rectangle is 54 square meters. If the width is $\frac{2}{3}$ of the length, then what is the perimeter of the rectangle in meters?

    (A) 15 (B) $\boxed{30}$ (C) 35 (D) 42 (E) None of these

11. A rhombus is inscribed in triangle $\triangle ABC$ in such a way that one of its vertices is A, two of its sides lie along $\overline{AB}$ and $\overline{AC}$, and its fourth vertex lies on $\overline{BC}$. If $AC = 6$ inches, $AB = 12$ inches and $BC = 8$ inches, the perimeter of the rhomus, in inches, is:

    (A) 12 (B) $\boxed{16}$ (C) 20 (D) 24 (E) None of these

12. A cone has a height that is three times its radius. If the volume of the cone is 125π, what is the radius of the cone?

    (A) $\boxed{5}$ (B) $5\sqrt[3]{2}$ (C) $10\sqrt[3]{2}$ (D) 15 (E) None of these

13. In the right triangle $\triangle ABC$ shown, D is on $\overline{AC}$, $m\angle BAD = 30°$, $m\angle BDC = 60°$ and $AD = 4$. Find BC.

(**A**) 2   (**B**) $2\sqrt{2}$ (**C**) $3\sqrt{2}$ (**D**) $\boxed{2\sqrt{3}}$ (**E**) None of these

14. The point $(2, 1)$ is reflected about the line $y = 2x$. What are the coordinates of the resulting point?

(**A**) $(-0.5, 2)$ (**B**) $(-0.3, 2.1)$ (**C**) $(-0.2, 2.8)$ (**D**) $\boxed{(-0.4, 2.2)}$ (**E**) None of these

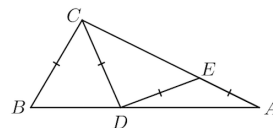**15.** Which of the following statements is false?

(**A**) $\boxed{\text{A parallelogram is a trapezoid.}}$ (**B**) A square is a parallelogram.

(**C**) A kite is a quadrilateral.(**D**) A rectangle is a parallelogram.(**E**) Some rhombi are rectangles.

16. A circle and a parabola are drawn on a piece of paper. What is the maximum number of regions they divide the paper into?

(**A**) 2   (**B**) 4   (**C**) 6   (**D**) 8   (**E**) $\boxed{\text{None of these=11}}$

17. Triangle $\triangle ABC$ is shown divided into three isosceles triangles, $\triangle BCD$, $\triangle CDE$ and $\triangle DEA$. Given that $BC = CD$, $CD = DE$, $DE = EA$ and $m\angle A = 20°$, find $m\angle B$.

(**A**) $20°$ (**B**) $40°$ (**C**) $\boxed{60°}$ (**D**) $80°$ (**E**) None of these
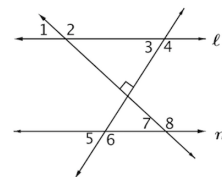
18. A segment has one endpoint of $(-3, 4)$ and a midpoint of $(2, 3)$. What are the coordinates of other endpoint?

(**A**) $\boxed{(7, 2)}$ (**B**) $\left(-\dfrac{1}{2}, \dfrac{7}{2}\right)$ (**C**) $(4, -3)$ (**D**) $\left(-2, -\dfrac{11}{2}\right)$ (**E**) None of these

19. In the following diagram, parallel lines ℓ and m are cut by two perpendicular transversals. Which of the following pairs of angles must be congruent?

I. $\angle 1$ and $\angle 3$ II. $\angle 2$ and $\angle 8$ III. $\angle 3$ and $\angle 5$

(**A**) I only   (**B**) $\boxed{\text{II and III}}$ (**C**) I and II

(**D**) I, II and III   (**E**) None of these

20. The diagonal of a square is 30. Find the area of the square.

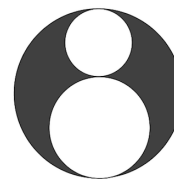(**A**) 225   (**B**) $\boxed{450}$ (**C**) 650   (**D**) 900   (**E**) None of these

21. The volume of a particular cylindrical can, which is made up of a square piece of metal and two circular disks, each with diameter k, is:

(**A**) $k^3\pi$ (**B**) $\dfrac{\pi k^2}{2}$ (**C**) $\dfrac{k^2(\pi + 2)}{2}$ (**D**) $\boxed{\dfrac{k^3\pi^2}{4}}$ (**E**) None of these

22. Given a regular hexagon, find the sum of its interior angles plus the sum of its exterior angles.

(**A**) $900°$ (**B**) $\boxed{1080°}$ (**C**) $1260°$ (**D**) $1440°$ (**E**) None of these

23. Two circles of radii 2 and 3 are externally tangent. A third circle is circumscribed about the two smaller circles. A pin is dropped inside the largest circle. What is the probability it lands in the shaded area?

    (A) 0.46   **(B)** $\boxed{0.48}$ **(C)** 0.50   **(D)** 0.52   **(E)** None of these

24. The perimeter of a rhombus is 10. What is the area of the rhombus?

    (A) $\dfrac{30}{13}$ **(B)** $\dfrac{25}{4}$ **(C)** 25   **(D)** $\dfrac{27}{13}$ **(E)** $\boxed{\text{Not enough information}}$
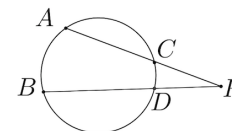
25. If the measures of all but one of the angles of a convex polygon sum to 680 degrees, what is the measure of the remaining angle in degrees?

    (A) 20   **(B)** $\boxed{40}$ **(C)** 70   **(D)** 140   **(E)** None of these

26. Which word best describes a triangle formed by sides in the ratio 6:10:18?

    (A) Scalene   **(B)** Right   **(C)** Obtuse   **(D)** Isosceles   **(E)** $\boxed{\text{Impossible}}$

27. In the figure shown, C is on $\overline{AP}$, D is on $\overline{PB}$, $PC = 3$, $AC = 5$ and $BD = 10$. Find PD.

    (A) 1.5   **(B)** $\boxed{2}$ **(C)** 3   **(D)** 6   **(E)** None of these

28. Triangle $\triangle ABD$ has right angle at $\angle B$. On $\overline{AD}$ there is a point C for which $AC = CD$ and $AB = BC$. If $BC = 3$, find the measure of angle $\angle BAD$.

    (A) $36°$ **(B)** $45°$ **(C)** $72°$ **(D)** $90°$ **(E)** $\boxed{\text{None of these}}$
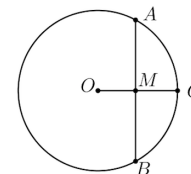
29. In the right triangle $\triangle ABC$ with right angle at $\angle B$, $AB = 8$ and $BC = 15$. Two arcs of circles are drawn: one with center C and radius 15, and the other with center A and radius 8. The arcs intersect the hypotenuse at points D and E, respectively. Find the length of $\overline{DE}$.

    (A) $\boxed{6}$ **(B)** 10   **(C)** 12   **(D)** 23   **(E)** None of these

30. The five angles of a convex pentagon have measure $2x$, $8x - 5$, $7x$, $5x - 20$ and $6x + 5$, respectively. Find the measure of the largest angle.

    (A) $108°$ **(B)** $140°$ **(C)** $\boxed{155°}$ **(D)** $230°$ **(E)** None of these

31. In the circle centered at point O, chord $\overline{AB}$ is the perpendicular bisector of radius $\overline{OC}$. If $AB = 8\sqrt{3}$, what is the area of the circle?

    (A) 36π **(B)** 48π **(C)** $\boxed{64\pi}$ **(D)** 96π **(E)** None of these

32. A regular hexagon $ABCDEF$ has sides 6 cm long. Determine the area of the triangle $\triangle ACE$ formed by connecting alternate vertices of the hexagon.

    (A) $9\sqrt{3}$ cm^2 **(B)** $18\sqrt{3}$ cm^2 **(C)** $\boxed{27\sqrt{3} \text{ cm}^2}$ **(D)** $36\sqrt{3}$ cm^2 **(E)** None of these

33. One rainy afternoon, you decide you want to start a craft project: make the Deathly Hallows (pictured) out of metal wire. The Deathly Hallows consists of a circle inscribed in an equilateral triangle with a single altitude. If you want a side of the triangle to be 2 inches, how much wire will it take to make the Deathly Hallows, in inches?

(**A**) $\dfrac{18 + 3\sqrt{3} + \pi}{3}$ (**B**) $\boxed{\dfrac{18 + 3\sqrt{3} + 2\sqrt{3}\pi}{3}}$

(**C**) $\dfrac{21 + \pi}{3}$ (**D**) $\dfrac{21 + 2\sqrt{3}\pi}{3}$ (**E**) None of these

34. A line containing points $(2, 1)$ and $(-8, m)$ is parallel to the line containing points $(7, m + 1)$ and $(11, 1)$. Find the value of m.

(**A**) $\boxed{-\dfrac{2}{3}}$ (**B**) 0   (**C**) $\dfrac{1}{6}$ (**D**) 3   (**E**) None of these

35. Consider trapezoid $ABCD$ with $\overleftrightarrow{DC}$ parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ perpendicular to $\overleftrightarrow{AB}$. Let $DC = 2$, $DA = 8$ and $AB = 20$. If P is a point on $\overline{AB}$ so that the area of quadrilateral $APCD$ equals the area of triangle $\triangle CPB$, then find PB.

(**A**) 9   (**B**) 10   (**C**) $\boxed{11}$ (**D**) 12   (**E**) None of these

36. The ratio of the measures of two angles is 3 to 4, and their sum is $84°$. What is the measure of the larger angle?
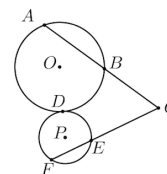
(**A**) $36°$ (**B**) $42°$ (**C**) $45°$ (**D**) $\boxed{48°}$ (**E**) None of these

37. The hypotenuse of a right triangle is 17 inches long and the area of the triangle is 60 square inches. Determine the sum of the other two sides, in inches.

(**A**) 20   (**B**) 22   (**C**) $\boxed{23}$ (**D**) 25   (**E**) None of these
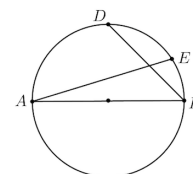
$\boxed{\text{Problem \#38 thrown out. Needs to specify that } \overline{CD} \text{ is tangent to both circles.}}$

38. In the figure shown, the circle centered at point O is tangent to the circle centered at point P at the point D, B is on $\overline{AC}$, and E is on $\overline{CF}$. Let $BC = 5$, $AB = 11$ and $CE = 8$. Find CD.

(**A**) 2   (**B**) $\sqrt{55}$ (**C**) 8   (**D**) $4\sqrt{5}$ (**E**) None of these

39. In the figure shown, $\overline{AB}$ is a diameter of the circle. If $m\angle EAB = 20°$, and $m\angle ABD = 50°$, determine the measure of the arc $\overset{\frown}{DE}$.
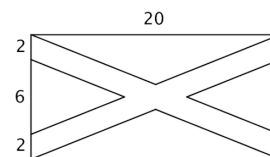
(**A**) $\boxed{40°}$ (**B**) $60°$ (**C**) $90°$ (**D**) $110°$ (**E**) None of these

40. Let $A = (0, 0)$, $B = (1, 4)$, $C = (3, -0.75)$, $D = (0, 3)$, and $E = (4, 0)$ be the vertices of triangles $\triangle ABC$ and $\triangle ADE$. What is the relationship between $\triangle ABC$ and $\triangle ADE$?

(**A**) $\triangle ABC \cong \triangle ADE$ (**B**) $\triangle ABC \cong \triangle AED$

(**C**) $\triangle ABC \sim \triangle ADE$ (**D**) $\boxed{\triangle ABC \sim \triangle AED}$ (**E**) None of these
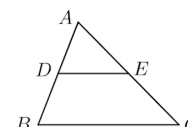
41. Amelia's front yard, which measures 20 feet wide by 10 feet long, is to have two diagonal concrete sidewalks put in, as shown below. If the sidewalks measure 2 feet along the side of the yard, how much of Amelia's yard will not be concrete?

(**A**) 50 ft^2 (**B**) $\boxed{125 \text{ ft}^2}$ (**C**) $200 - 8\sqrt{29} \text{ ft}^2$

(**D**) $200 - 20\sqrt{5} \text{ ft}^2$ (**E**) None of these

42. A regular square pyramid has height $\sqrt{15}$ with the area of a triangular face equal to 10. Find the surface area of the pyramid.

(**A**) $40 + \sqrt{15}$ (**B**) $40 + 2\sqrt{15}$ (**C**) $\boxed{60}$ (**D**) 120   (**E**) None of these

43. In $\triangle ABC$ shown, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. If $BC = 7x + 1$ and $DE = 4x - 2$, then find the length BC.
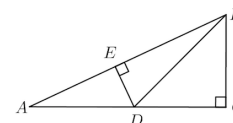
(**A**) 10   (**B**) 18   (**C**) 22   (**D**) $\boxed{36}$ (**E**) None of these

44. If 5 and 9 are two sides of a right triangle, which of the following could be the third side?

(**A**) $\sqrt{24}$ (**B**) 5   (**C**) 7   (**D**) $\boxed{\sqrt{56}}$ (**E**) None of these

45. In $\triangle ABC$ shown, $\angle ACB$ and $\angle BED$ are right angles, $m\angle EAD = 25°$ and $m\angle BDC = 40°$. If $BC = 4$, find ED.

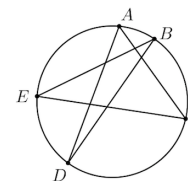(**A**) $\boxed{\dfrac{4\sin 15°}{\sin 40°}}$ (**B**) $\dfrac{4\sin 75°}{\sin 50°}$ (**C**) $\dfrac{\sin 40°}{4\sin 15°}$ (**D**) $\dfrac{\sin 50°}{4\sin 75°}$ (**E**) 2

46. In $\triangle ABC$ with right angle at C, altitude $\overline{CH}$ and median $\overline{CM}$ trisect the right angle. If the area of $\triangle CHM$ is K, then the area of $\triangle ABC$ is:

(**A**) $6K$ (**B**) $4\sqrt{3}K$ (**C**) $3\sqrt{3}K$ (**D**) $3K$ (**E**) $\boxed{4K}$

47. Points A, B, C, D and E are drawn on a circle and connected to form a five pointed star. Find the value of $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$.

(**A**) $120°$ (**B**) $\boxed{180°}$ (**C**) $240°$ (**D**) $360°$ (**E**) None of these

48. In $\triangle ABC$ with right angle at C, midpoints D and E lie along $\overline{AC}$ and $\overline{BC}$, respectively. If $AC = 6$ and $BC = 8$, find the height of trapezoid $ABED$.

(**A**) $\boxed{2.4}$ (**B**) 3.5 (**C**) $\sqrt{5}$ (**D**) $\sqrt{7}$ (**E**) None of these

49. ABCD is a square with side of length 1. Points E and F are taken respectively on sides $\overline{AB}$ and $\overline{AD}$ so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. What is the maximum area of $CDFE$?

(**A**) $\boxed{\dfrac{5}{8}}$ (**B**) $\dfrac{23}{32}$ (**C**) $\dfrac{7}{8}$ (**D**) $\dfrac{9}{10}$ (**E**) None of these

50. The angles of a convex, irregular pentagon form an arithmetic sequence. If all angles have integer measure, what is the largest measure that the smallest angle can have in degrees?

(**A**) 100   (**B**) 106   (**C**) 107   (**D**) 110   (**E**) $\boxed{\text{None of these}=108}$