The 35th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 27, 2016 at Regional Testing Centers
Second Round: April 9, 2016 at The University of North Alabama

GEOMETRY EXAM

Construction of this test directed
by
Scott H. Brown and Luke A. Smith, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Editing by Miranda Bowie and Ashley Johnson, The University of North Alabama
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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

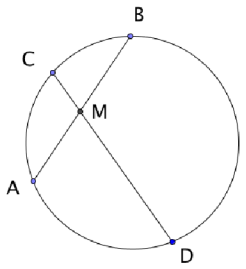
1. What is the equation of the line that goes through the point $(2, -5)$ and is parallel to $x - 3y = 6$?

(**A**) $3y = x - 13$ (**B**) $3y = x - 11$ (**C**) $\boxed{3y = x - 17}$ (**D**) $3y = x - 19$ (**E**) None of these

2. Find the volume of a cube with a surface area of 96 square feet.

(**A**) $\boxed{64 \text{ ft}^3}$ (**B**) 72 ft^3 (**C**) 88 ft^3 (**D**) 96 ft^3 (**E**) None of these

3. In the figure shown, chord $\overline{AB}$ has a length of 18 and is bisected by the chord $\overline{CD}$. What is the length of $\overline{CD}$ if $\overline{DM}$ is five times as long as $\overline{CM}$?



(**A**) $\dfrac{44\sqrt{3}}{3}$ (**B**) 22 (**C**) $\boxed{\dfrac{54\sqrt{5}}{5}}$ (**D**) 24 (**E**) None of these

4. Given that the points $(1, 4)$, $(6, 12)$ and $(c, 10)$ are collinear, what is the value of c?

(**A**) 3.5 (**B**) 4 (**C**) 4.25 (**D**) $\boxed{4.75}$ (**E**) None of these

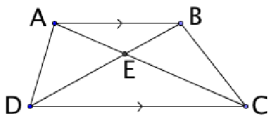
5. When the circumference of a sphere is increased from 20 meters to 25 meters, the radius is increased by

(**A**) $\boxed{\dfrac{5}{2\pi} \text{ m}}$ (**B**) 5 m (**C**) $\dfrac{5}{\pi}$ m (**D**) $\dfrac{5}{2}$ m (**E**) None of these

6. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the regular hexagon is $6\sqrt{3}$, then what is the area of the equilateral triangle?

(**A**) $\boxed{4\sqrt{3}}$ (**B**) $6\sqrt{3}$ (**C**) $8\sqrt{3}$ (**D**) $2\sqrt{3}$ (**E**) None of these

7. In a trapezoid $ABCD$ with $\overleftrightarrow{AB}$ parallel to $\overleftrightarrow{CD}$, the diagonals intersect at a point E. The area of triangle $\triangle ABE$ is 32 and of triangle $\triangle CDE$ is 50. Find the area of the trapezoid.



(**A**) 136 (**B**) $\boxed{162}$ (**C**) 178 (**D**) 184 (**E**) None of these

8. Name the polygon that satisfies all of the following characteristics:

I. 4 Sides
II. Opposite Angles Congruent
III. Diagonals are perpendicular bisectors

(**A**) Kite (**B**) $\boxed{\text{Rhombus}}$ (**C**) Parallelogram (**D**) Trapezoid (**E**) Rectangle

9. A 3 by 3 by 3 cube is formed by stacking 1 by 1 by 1 cubes. How many cubes with sides of integer length are contained in the 3 by 3 by 3 cube?

(**A**) 18 (**B**) 27 (**C**) $\boxed{36}$ (**D**) 50 (**E**) None of these

10. Find the radius of the circle inscribed in a triangle with sides 5, 12 and 13 in length.

(**A**) $\dfrac{3}{2}$ (**B**) $\boxed{2}$ (**C**) $\dfrac{5}{2}$ (**D**) 3 (**E**) None of these

11. The central angle of a regular decagon is

(**A**) $18°$ (**B**) $\boxed{36°}$ (**C**) $52°$ (**D**) $144°$ (**E**) None of these

12. Two tangents to the same circle form a $50°$ angle. The radius of the circle is 10. Find the length of the smaller of the intercepted arcs.

(**A**) $\boxed{\frac{65\pi}{9}}$ (**B**) $\frac{40\pi}{9}$ (**C**) $\frac{25\pi}{9}$ (**D**) $\frac{10\pi}{9}$ (**E**) None of these

13. The diameter of a cone is 10 and its slant height is 13. What is its volume?

(**A**) $\boxed{100\pi}$ (**B**) $\frac{175\pi}{3}$ (**C**) 400π (**D**) $\frac{700\pi}{3}$ (**E**) None of these

14. A square with side length 2 is drawn in the Cartesian Plane with vertices $(1,0), (1,2), (-1,2)$, and $(-1,0)$. Which of the following is a Line of Symmetry of the square?

I. $y = x$
II. $x = 1$
III. $y = 1$
IV. $y = 1 - x$

(**A**) II and IV (**B**) I and II (**C**) $\boxed{\text{III and IV}}$ (**D**) I only (**E**) II and III

15. The measure of each interior angle of a regular polygon is 8 times that of an exterior angle of the polygon. Find the number of sides of the polygon.

(**A**) 14 (**B**) $\boxed{18}$ (**C**) 22 (**D**) 24 (**E**) None of these

16. The area of a square inscribed in a circle is 30 units. Determine the perimeter of the square inscribed in a semicircle of the same radius.

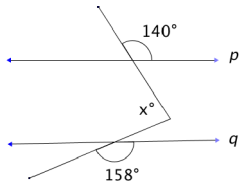
(**A**) $4\sqrt{15}$ (**B**) $4\sqrt{30}$ (**C**) $\boxed{8\sqrt{3}}$ (**D**) $8\sqrt{15}$ (**E**) None of these

17. Ray $\overrightarrow{OB}$ bisects $\angle AOC$. If $m\angle AOB = 2x + 10$ and $m\angle BOC = 8x - 14$, what is $m\angle AOC$?

(**A**) $\boxed{36°}$ (**B**) $42°$ (**C**) $54°$ (**D**) $72°$ (**E**) None of these

18. Assume $p \parallel q$ in the figure shown. Find the angle supplementary to angle x.

(**A**) $112°$ (**B**) $122°$ (**C**) $128°$ (**D**) $138°$ (**E**) $\boxed{\text{None of these } (118°)}$



19. Two circular gears are tangent to each other. The centers are fixed and the radii are 60 feet and 80 feet, respectively. How many revolutions has the larger gear made when the smaller gear has made 6 revolutions?

(**A**) 8 (**B**) $\frac{3}{2}$ (**C**) 6 (**D**) $\boxed{\frac{9}{2}}$ (**E**) None of these

20. The four angles of a quadrilateral form an arithmetic sequence. The largest is 15 degrees less than twice the smallest. What is the degree measure of the largest angle?

(**A**) $75°$ (**B**) $85°$ (**C**) $\boxed{115°}$ (**D**) $125°$ (**E**) None of these

21. A rectangular picture of width 20 inches and height of 10 inches is framed in a rectangular frame 3 inches wide. What is the area of the frame?

(**A**) $\boxed{216 \text{ in}^2}$ (**B**) 212 in^2 (**C**) 194 in^2 (**D**) 182 in^2 (**E**) None of these

22. Given that it is 3 o'clock, exactly how long will it take for the minute hand to catch up with the hour hand?

(**A**) 15 min (**B**) $\boxed{16\frac{4}{11} \text{ min}}$ (**C**) $16\frac{1}{2}$ min (**D**) 17 min (**E**) None of these

23. What is the total surface area of a right circular cylinder whose radius is 6 units and whose height is 10 units?

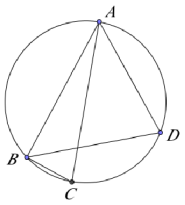
(**A**) $156\pi \text{ u}^2$ (**B**) $172\pi \text{ u}^2$ (**C**) $188\pi \text{ u}^2$ (**D**) $\boxed{192\pi \text{ u}^2}$ (**E**) None of these

24. A parallelogram's vertices have coordinates of $(0, 0)$, $(1, \sqrt{3})$, $(4, 0)$ and $(5, \sqrt{3})$. What is the area of the parallelogram?

(**A**) $2\sqrt{3}$ (**B**) 4 (**C**) $\boxed{4\sqrt{3}}$ (**D**) 8 (**E**) None of these

25. Points A, B, C and D lie on a circle with $\overline{AC}$ a diameter, $AB = 4$ and $BC = 2$, $\angle ABD \cong \angle CBD$. What is BD?

(**A**) $2\sqrt{3}$ (**B**) $5\sqrt{2}$ (**C**) $3\sqrt{3}$ (**D**) $\boxed{3\sqrt{2}}$ (**E**) None of these



26. If the altitude of an equilateral triangle is $\sqrt{6}$, then what is its area?

(**A**) $\boxed{2\sqrt{3}}$ (**B**) $6\sqrt{3}$ (**C**) $2\sqrt{6}$ (**D**) $4\sqrt{3}$ (**E**) None of these

27. For what value of k are the lines $2x + 3y = 4k$ and $x - 2ky = 7$ perpendicular?

(**A**) $-\frac{3}{4}$ (**B**) $\frac{1}{6}$ (**C**) $\boxed{\frac{1}{3}}$ (**D**) $\frac{1}{2}$ (**E**) None of these

28. A rectangular field is half as wide as it is long and is completely enclosed by x yards of fencing. The area in terms of x is

(**A**) $2x^2$ (**B**) $\boxed{\frac{x^2}{18}}$ (**C**) $4x^2$ (**D**) $\frac{x^2}{4}$ (**E**) None of these

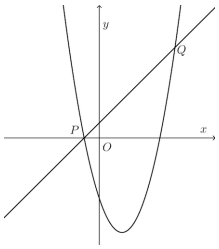
29. A regular hexagon is inscribed in a circle of radius 10. What is the area of the hexagon?

(**A**) $\boxed{150\sqrt{3}}$ (**B**) 150 (**C**) 600 (**D**) $300\sqrt{3}$ (**E**) None of these

30. A man who is 6 ft tall would like to determine the height of a building. He measures its shadow and finds it to be 28 ft long, while his own shadow is 3.5 ft long. How tall is the building?

(**A**) 98 ft (**B**) 75 ft (**C**) 56 ft (**D**) $\boxed{48 \text{ ft}}$ (**E**) None of these

31. In the figure shown, the line $y = x + 1$ intersects the parabola $y = x^2 - 3x - 4$ at points P and Q. What are the coordinates of point Q?



(**A**) $(-1, 0)$ (**B**) $(4, 0)$ (**C**) $(4, 5)$ (**D**) $\boxed{(5, 6)}$ (**E**) None of these

32. Eight times the reciprocal of the circumference of a circle equals the diameter of the circle. Find the area of the circle.

(**A**) 1 (**B**) $\boxed{2}$ (**C**) 4 (**D**) 8 (**E**) None of these

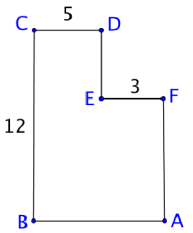
33. The area of a rectangle is 54 square feet. If its width is 2/3 of the length, then what is the perimeter of the rectangle?

(**A**) 18 ft (**B**) 20 ft (**C**) 25 ft (**D**) 27 ft (**E**) $\boxed{\text{None of these (30 ft)}}$

34. The smallest angle of a rhombus is one-half the measure of the larger angle. The shorter diagonal is 20 m. Find the perimeter of the rhombus.

(**A**) 40 m (**B**) 60 m (**C**) $\boxed{80 \text{ m}}$ (**D**) 100 m (**E**) None of these

35. In the polygon pictured, $BC = 12$, $CD = 5$, and $EF = 3$. All angles are right angles. Find the area of the polygon.



(**A**) 69 (**B**) 78 (**C**) 87 (**D**) 96 (**E**) $\boxed{\text{Not enough information}}$

36. What is the center of the circle described by the equation $x^2 + y^2 - 8x + 16y = -25$?

(**A**) $(4, 4)$ (**B**) $\boxed{(4, -8)}$ (**C**) $(8, 4)$ (**D**) $(0, 7)$ (**E**) None of these

37. Three circles of radii 2, 4 and 6 are tangent to each other externally. Find the area of the triangle formed by connecting their centers.

(**A**) 18 (**B**) 20 (**C**) 22 (**D**) $\boxed{24}$ (**E**) None of these

38. A rectangular box measures 2 feet by 4 feet by 5 feet. What is the length of the longest pole that can fit in the box?

(**A**) $\sqrt{5}$ (**B**) $5\sqrt{2}$ (**C**) $\boxed{3\sqrt{5}}$ (**D**) $5\sqrt{3}$ (**E**) None of these

39. A regular n-gon has interior angles each of measure $162°$. What is the value of n?

(**A**) 18 (**B**) $\boxed{20}$ (**C**) 22 (**D**) 24 (**E**) None of these

40. Which of the following is an intersection point of the line $y = 2x+1$ and the circle $(x-1)^2+(y+1)^2 = 16$?

(**A**) $\left(\frac{11}{5}, \frac{27}{5}\right)$ (**B**) $\left(\frac{11}{5}, \frac{3}{15}\right)$ (**C**) $(3, 7)$ (**D**) $\boxed{(1, 3)}$ (**E**) None of these

41. Quadrilateral $ABCD$ is inscribed in a circle with $m\angle C = 85°$. Find $m\angle A$.

(**A**) $85°$ (**B**) $\boxed{95°}$ (**C**) $105°$ (**D**) $115°$ (**E**) Not enough information

42. If the radius of a sphere is tripled, what is the ratio of the new volume to the old volume?

(**A**) $9:1$ (**B**) $\boxed{27:1}$ (**C**) $32:1$ (**D**) $64:1$ (**E**) None of these

43. Determine the measure of an angle in degrees such that the sum of its supplement and its complement is $156°$

(**A**) $33°$ (**B**) $\boxed{57°}$ (**C**) $66°$ (**D**) $124°$ (**E**) None of these

44. Find the area of an equilateral triangle whose vertices lie on a circle with radius 2 cm.

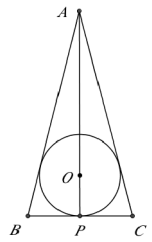
(**A**) 3π cm^2 (**B**) $\boxed{3\sqrt{3}\text{ cm}^2}$ (**C**) $\sqrt{3}\pi$ cm^2 (**D**) 6 cm^2 (**E**) None of these

45. Which of the following classical constructions is impossible using only a compass and a straight edge?

I. Draw the perpendicular bisector of a segment
II. Find the midpoint of a segment
III. Inscribe a regular hexagon in a circle
IV. Trisect an angle

(**A**) $\boxed{\text{IV Only}}$ (**B**) III and IV (**C**) I and III (**D**) II Only (**E**) I and II

46. In the figure shown, the circle is inscribed in isosceles triangle $\triangle ABC$, with segment $\overline{AP}$ passing through center O of the circle, $AC = AB = 12$ and $BP = 4$. Find the radius of the circle.



(**A**) $2\sqrt{3}$ (**B**) $4\sqrt{2}$ (**C**) $4\sqrt{3}$ (**D**) $\boxed{2\sqrt{2}}$ (**E**) None of these

47. Find the radius of the largest sphere that can fit entirely inside a rectangular box with dimensions of 5 in by 7 in by 11 in.

(**A**) $\frac{9}{2}$ (**B**) $\frac{7}{2}$ (**C**) $\boxed{\frac{5}{2}}$ (**D**) $\frac{3}{2}$ (**E**) None of these

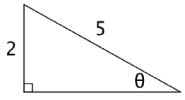
48. A particular brand of pepperoni pizza has 16 calories per square inch. If a pizza 14 inch in diameter is cut into 8 equal slices, how many calories are in one slice?

(**A**) 28 (**B**) 392π (**C**) $\boxed{98\pi}$ (**D**) 28π (**E**) None of these

49. A straight line joins the points $(3, 9)$ and $(-1, 1)$. Find the non-zero coordinate of the x-intercept of the line.

(**A**) $-\frac{5}{2}$ (**B**) -2 (**C**) 3 (**D**) $\boxed{-\frac{3}{2}}$ (**E**) None of these

50. For θ defined in the figure find $\cos\theta$



(**A**) $\frac{2}{5}$ (**B**) $\frac{5}{2}$ (**C**) $\boxed{\frac{\sqrt{21}}{5}}$ (**D**) $\frac{\sqrt{21}}{2}$ (**E**) None of these