The 34th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 28, 2015 at Regional Testing Centers
Second Round: April 11, 2015 at University of North Alabama

GEOMETRY EXAMINATION

Construction of this test directed
by
Scott H. Brown and Luke Smith, Auburn University Montgomery

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Editing by Miranda Bowie and Ashley Johnson, University of North Alabama
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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

The following information is courtesy of the U.S. Bureau of Labor Statistics.

- The median salary of a Mathematician in 2012 was $101,360 per year.
- Over the next 10 years, the job opportunities for mathematicians are expected to grow by 23%!

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. In triangle $\triangle ABC$, $m\angle C = 38°$ and $AB = AC$. Find $m\angle A$.

(**A**) $92°$ (**B**) $98°$ (**C**) $\boxed{104°}$ (**D**) $116°$ (**E**) None of these.

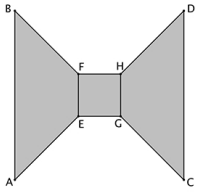
2. The area of a given rhombus is 154 u^2. If one of the diagonals has length 14 u, what is the length of the other diagonal?

(**A**) 11 u (**B**) 16 u (**C**) 18 u (**D**) $\boxed{22\text{ u}}$ (**E**) None of these.

3. In a circle, two chords $\overline{AB}$ and $\overline{DC}$ intersect at point O. If $AO = 2$ and $OC = DO = 4$, how long is $\overline{AB}$?

(**A**) $\boxed{10\text{ u}}$ (**B**) $2\sqrt{5}$ u (**C**) $\sqrt{38}$ u (**D**) 8 u (**E**) None of these.

4. In the figure to the right, $m\angle A = m\angle B = m\angle C = m\angle D = 45°$, $AE = BF = HD = CG$, $AB = CD$, and $EFHG$ is a square. How many lines of reflection symmetry does the figure have?



(**A**) 1 (**B**) $\boxed{2}$ (**C**) 3 (**D**) 4 (**E**) None of these.

5. The perimeter of a particular square and regular hexagon are equal. What is the ratio of the length of a side of the regular hexagon to the length of a side of the square?

(**A**) $\boxed{\frac{2}{3}}$ (**B**) $\frac{3}{4}$ (**C**) $\frac{\sqrt{3}}{2}$ (**D**) $\frac{3}{2}$ (**E**) None of these.

6. What is the length of the largest pole that can be placed in a rectangular box with dimensions 2 x 3 x 4?

(**A**) $3\sqrt{2}$ (**B**) $\frac{7}{2}$ (**C**) 9 (**D**) $\boxed{\sqrt{29}}$ (**E**) None of these.

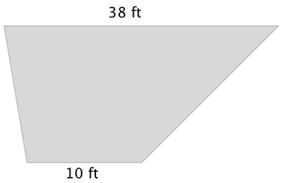
7. Which of the following could not be the sides of a triangle?

(**A**) $7, 7, 8$ (**B**) $19, 21, 24$ (**C**) $5, 12, 13$ (**D**) $3, 4, 6$ (**E**) $\boxed{2, 7, 11}$

8. Two sides of a right triangle are the roots of $x^2 - 8x + 15$. If the sides are all natural numbers, what is the area of the triangle?

(**A**) $\boxed{6\text{ u}}$ (**B**) 7 u (**C**) 8 u (**D**) 12 u (**E**) None of these.

9. A dam is constructed with a trapezoidal cross-section measuring 38 feet at the top and 10 feet at the bottom. What is the average width of the dam?



(**A**) 14 ft (**B**) 18 ft (**C**) 20 ft (**D**) $\boxed{24\text{ ft}}$ (**E**) None of these.

10. Three cubes of volume 1, 8 and 64 respectively are glued together. What is the smallest possible surface area of the resulting configuration?

(**A**) 96 (**B**) 108 (**C**) $\boxed{114}$ (**D**) 120 (**E**) None of these.

11. How many vertices does a regular polyhedron with 16 faces and 32 edges have?

(**A**) 14 (**B**) 16 (**C**) $\boxed{18}$ (**D**) 24 (**E**) None of these.

12. Given that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\angle A \cong \angle D$, we can conclude that $\triangle ABC$ and $\triangle DEF$ are

(**A**) similar. (**B**) congruent. (**C**) both isosceles.

(**D**) both acute. (**E**) $\boxed{\text{Not enough information is given to draw a conclusion.}}$

13. Two congruent circles in the same plane cannot have which of the following number of common tangents:

(**A**) $\boxed{1}$ (**B**) 2 (**C**) 3 (**D**) 4 (**E**) None of these.

14. Find the length of the line segment between the coordinates $(6, 5)$ and $(3, -1)$.

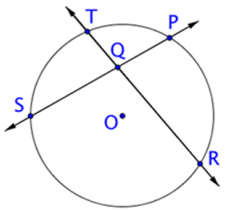
(**A**) $3\sqrt{2}$ (**B**) 6 (**C**) $\sqrt{42}$ (**D**) $\boxed{3\sqrt{5}}$ (**E**) None of these.

15. Find the area of the segment whose arc is $60°$ in a circle of radius 6.

(**A**) $6\pi - \sqrt{3}$ (**B**) $\boxed{6\pi - 9\sqrt{3}}$ (**C**) $6\pi - 3\sqrt{3}$ (**D**) $6\pi - 8\sqrt{3}$ (**E**) None of these.

16. In the given figure, $m\widehat{PR} = 75°$ and $m\widehat{TS} = 45°$. Find $m\angle PQR$.

(**A**) $55°$ (**B**) $\boxed{60°}$ (**C**) $75°$ (**D**) $90°$ (**E**) None of these.



17. In triangle $\triangle ABC$, the interior angle at vertex B has measure $72°$, and the exterior angle at vertex A has measure $145°$. Find the measure of the exterior angle at vertex C.

(**A**) $72°$ (**B**) $77°$ (**C**) $\boxed{107°}$ (**D**) $135°$ (**E**) None of these.

18. A chord of length 24 cm is drawn in a circle of radius 13 cm. Find the distance from the chord to the center of the circle.

(**A**) $\boxed{5}$ (**B**) $6\sqrt{3}$ (**C**) 6 (**D**) $3\sqrt{5}$ (**E**) None of these.

19. What is the measure of the central angle of a 15-sided regular polygon in degrees?

(**A**) $12°$ (**B**) $156°$ (**C**) $\boxed{24°}$ (**D**) $108°$ (**E**) None of these.

20. Suppose six tennis balls are stacked tightly in a cylindrical container whose diameter is approximately the same as a tennis ball's. If the balls are stacked to the top of the container, find the fraction of the container that is occupied by the tennis balls.

(**A**) $\frac{1}{4}$ (**B**) $\boxed{\frac{2}{3}}$ (**C**) $\frac{3}{4}$ (**D**) $\frac{5}{6}$ (**E**) None of these.

21. The area of a particular equilateral triangle is equal to its perimeter. What is the length of a side of this triangle?

(**A**) $\sqrt{3}$ (**B**) $2\sqrt{3}$ (**C**) $\boxed{4\sqrt{3}}$ (**D**) $6\sqrt{3}$ (**E**) None of these.

22. A line through the points $(M, -9)$ and $(7, M)$ has slope M. What is the equation of the line that goes through these two points?

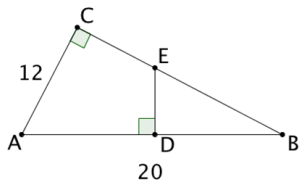
(**A**) $y = 3x - 24$ (**B**) $y = 6x + 18$ (**C**) $y = 4x - 16$ (**D**) $y = 6x - 18$ (**E**) $\boxed{y = 3x - 18}$

23. A ball is placed in a cubical box touching each side of the box. The volume of the ball is 288π cubic meters. Find the length of a side of the box.

(**A**) 6 m (**B**) 8 m (**C**) $\boxed{12 \text{ m}}$ (**D**) 14 m (**E**) None of these.

24. Given right triangle $\triangle ABC$, $\overline{AD} \cong \overline{DB}$, $\overline{DE} \perp \overline{AB}$, $AB = 20$ and $AC = 12$, find the area of the quadrilateral $ADEC$.

(**A**) $\boxed{58.5}$ (**B**) 56 (**C**) 45 (**D**) 37.5 (**E**) None of these.



25. The number of diagonals that can be drawn in a polygon of 30 sides is:

(**A**) 380 (**B**) 395 (**C**) 400 (**D**) $\boxed{405}$ (**E**) None of these.

26. A farmer has 12,000 feet of fencing. He wants to fence in an area 3 times as long as it is wide for the rectangular plot of land. What is the width of this area if he uses all of the fencing?

(**A**) $\boxed{1500 \text{ ft}}$ (**B**) 1600 ft (**C**) 1700 ft (**D**) 1800 ft (**E**) None of these.

27. Given a circle centered at (3,4) that passes through (7,1), which of the following is the equation of the tangent line to the circle at the point (7,1)?

(**A**) $3x - 4y = 17$ (**B**) $\boxed{4x - 3y = 25}$ (**C**) $3x + 4y = 25$ (**D**) $4x + 3y = 31$ (**E**) $4x - 3y = 17$

28. If the length of a diagonal of a square is $x + y$, what is the area of the square?

(**A**) $(x + y)^2$ (**B**) $x^2 + y^2$ (**C**) $\boxed{\frac{1}{2}(x + y)^2}$ (**D**) $2x + 2y$ (**E**) $\frac{1}{2}(x^2 + y^2)$

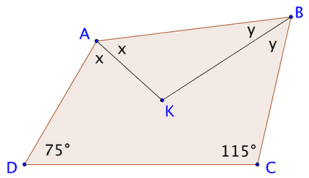
29. A jar contains 50 cubic inches of air. On each stroke of a pump, 20% of the area is removed from the jar. How much air is left after the second stroke?

(**A**) 8 (**B**) 16 (**C**) 24 (**D**) $\boxed{32}$ (**E**) None of these.

30. The points $(4, 3)$, $(2, -3)$, and $(5, \dfrac{k}{2})$ are on the same straight line. The value(s) of k is (are):

(**A**) 6 (**B**) $\boxed{12}$ (**C**) $-8, 8$ (**D**) $6, 12$ (**E**) None of these.

31. In the diagram shown, $m\angle D = 75°$ and $m\angle C = 115°$. There is a point K interior to quadrilateral $ABCD$ such that $\overline{AK}$ bisects angle $\angle A$ and $\overline{BK}$ bisects $\angle B$. Find the measure of $\angle AKB$.



(**A**) $75°$ (**B**) $85°$ (**C**) $\boxed{95°}$ (**D**) $105°$ (**E**) None of these.

32. A rectangular piece of paper is folded in half three times in succession. The final folded piece has an area of x square inches. If the area of the unfolded piece of paper is 320 square inches, find x.

(**A**) $\boxed{40 \text{ in}^2}$ (**B**) 60 in^2 (**C**) 80 in^2 (**D**) 50 in^2 (**E**) None of these.

33. Determine the equation of the line passing through the points of intersection of the circles $(x+2)^2 + (y+1)^2 = 9$ and $(x+3)^2 + (y+4)^2 = 16$.

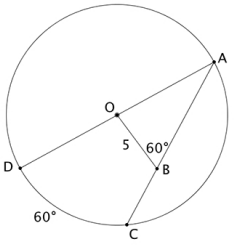
(**A**) $\boxed{2x + 6y = -13}$ (**B**) $10x - 6y = -7$

(**C**) $-2x + 6y = -7$ (**D**) $-10x + 6y = 25$ (**E**) $2x - 6y = 13$

34. A square $ABCD$ of side length 1 has a point E on $\overline{CD}$ and point F in the interior of the square so that $\overline{EF} \perp \overline{DC}$ and $\overline{AF} \cong \overline{BF} \cong \overline{EF}$. Determine the area of $AFBCD$.

(**A**) $\frac{9}{16}$ (**B**) $\frac{7}{8}$ (**C**) $\boxed{\frac{13}{16}}$ (**D**) $\frac{5}{8}$ (**E**) None of these.

35. In the circle shown, $\overline{AD}$ is a diameter, $\overline{ABC}$ is a chord, $BO = 5$ and $m\angle ABO = m\overset{\frown}{CD} = 60°$. Find the length of $\overline{BC}$.



(**A**) $3\sqrt{5}$ (**B**) $\boxed{5}$ (**C**) $5\sqrt{3}$ (**D**) 10 (**E**) None of these.

36. If the measures of an angle and its complement are in ratio of 2:7, what is the measure of the angle's supplement, in degrees?

(**A**) $110°$ (**B**) $130°$ (**C**) $150°$ (**D**) $\boxed{160°}$ (**E**) None of these.

37. As the number of sides of a polygon increase from 3 to n, the sum of the exterior angles will always:

(**A**) decrease (**B**) equal $180°$ (**C**) increase (**D**) $\boxed{\text{equal } 360°}$ (**E**) equal $(n-2)180°$

38. A sphere has volume W. If its radius is doubled, the volume of the resulting sphere is $V = kW$. What is the value of k?

(**A**) 16 (**B**) $\boxed{8}$ (**C**) 4 (**D**) 2 (**E**) None of these.

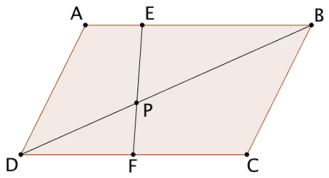
39. If the area of a square inscribed in a circle is 30 square centimeters, what is the perimeter of a square inscribed in a semicircle of the same circle?

(**A**) $\boxed{8\sqrt{3}}$ (**B**) $6\sqrt{3}$ (**C**) $4\sqrt{3}$ (**D**) $2\sqrt{3}$ (**E**) None of these.

40. The number of square feet in the total surface area of a right circular cylinder is equal to the number of cubic feet in its volume. If the radius of its base is five times its altitude, what is the length of the radius?

(**A**) $\boxed{12}$ (**B**) 8 (**C**) 16 (**D**) 10 (**E**) None of these.

41. Parallelogram $ABCD$ shown is such that $AE = \frac{1}{4}AB$, F is the midpoint of $\overline{DC}$ and $\overline{EF}$ cuts $\overline{BD}$ at P. Find the ratio of DP to BP.

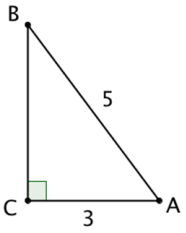


(**A**) 1 : 3 (**B**) $\boxed{2:3}$ (**C**) 2 : 5 (**D**) 3 : 4 (**E**) None of these.

42. A parabola $y = ax^2 + bx + c$ has vertex $(4, 2)$. If $(6, 0)$ is on the parabola, what is the product ac?

(**A**) 4 (**B**) -2 (**C**) $-\frac{1}{2}$ (**D**) $\boxed{3}$ (**E**) None of these.

43. Using right triangle $\triangle ABC$, find the value of $\cos A - \tan B$.



(**A**) $\boxed{-\frac{3}{20}}$ (**B**) $\frac{7}{15}$ (**C**) $-\frac{11}{15}$ (**D**) $\frac{1}{20}$ (**E**) None of these.

44. What is the area of the circle which passes through the three points $(2, 0)$, $(0, 0)$ and $(1, 1)$?

(**A**) 2π (**B**) 4π (**C**) $\boxed{\pi}$ (**D**) 3π (**E**) None of these.

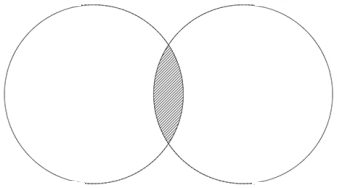
45. The perimeter of a right triangle is $12 + 8\sqrt{3}$. The sum of the squares of the three sides is 294. Find the length of the hypotenuse.

(**A**) $\boxed{7\sqrt{3}}$ (**B**) $8\sqrt{3}$ (**C**) $7\sqrt{6}$ (**D**) $4\sqrt{3}$ (**E**) None of these.

46. The volume of a cube (in cubic meters) added to 12 times the sum of the lengths of its edges is equal to 4 times its surface area (in square meters). Find the length of a side of the cube.

(**A**) $\boxed{12}$ (**B**) $8\sqrt{3}$ (**C**) 6 (**D**) $12\sqrt{3}$ (**E**) None of these.

47. In the figure shown, each of the circles has a diameter of 12 and the distance between the intersection points is 6. Find the area of the overlapping region.



(**A**) $8\pi - 6\sqrt{3}$ (**B**) $10\pi - 16\sqrt{3}$ (**C**) $\boxed{12\pi - 18\sqrt{3}}$

(**D**) $6\pi - 9\sqrt{3}$ (**E**) None of these.

48. Find the height of a pyramid with a square base whose area is 16 square inches and whose lateral edges have length 5 inches.

(**A**) 4 (**B**) $\boxed{\sqrt{17}}$ (**C**) $3\sqrt{2}$ (**D**) 4.5 (**E**) None of these.

49. When the circumference of a spherical balloon is increased from 20 inches to 25 inches, the radius is increased by:

(**A**) $\frac{5}{\pi}$ in (**B**) $\frac{5}{2}$ in (**C**) 5 in (**D**) $\boxed{\frac{5}{2\pi} \text{ in}}$ (**E**) None of these.

50. What is the perimeter of a right triangle with hypotenuse of 5 and area of 6?

(**A**) 8 (**B**) 9 (**C**) 11 (**D**) $\boxed{12}$ (**E**) None of these.