The 36th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 25, 2017 at Regional Testing Centers
Second Round: April 8, 2017 at The University of North Alabama

COMPREHENSIVE EXAM

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\overset{\frown}{AB}$ is the arc between A and B.
- If A and B are points on a circle, then $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. Simplify $\left[(81)^{3/4}\left(\frac{9}{25}\right)^{-3/2}+(153)^0\left(\frac{1}{7}\right)^{-1}(625)^{3/4}\right]^{-1/3}$.

(**A**) $\frac{1}{100}$ (**B**) $\frac{7+\sqrt[3]{7}}{35}$ (**C**) $\frac{7+\sqrt[3]{49}}{35}$ (**D**) $\boxed{\frac{1}{10}}$ (**E**) None of these

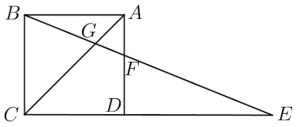
2. The function $f(x)=\dfrac{x}{x^2+1}$ is:

(**A**) $\boxed{\text{Odd}}$ (**B**) Even (**C**) Neither even nor odd (**D**) Both even and odd (**E**) None of these

3. A rectangle measuring 40 cm by 30 cm is inscribed in a circle. Find the circumference of the circle.

(**A**) 35π (**B**) 40π (**C**) 55π (**D**) 75π (**E**) $\boxed{\text{None of these} = 50\pi}$

4. $ABCD$ is a square as shown, with a line through B which intersects the extension of $\overline{CD}$ at point E, the side $\overline{AD}$ at point F and the diagonal $\overline{AC}$ at point G. If $BG=3$ and $GF=1$, find the length of $\overline{FE}$.



(**A**) 4 (**B**) $4\sqrt{2}$ (**C**) $\dfrac{8\sqrt{2}}{5}$ (**D**) $\boxed{8}$ (**E**) None of these

5. Your history teacher gives you a five question multiple choice quiz where each question has four possible answer choices. You forgot to study and are going to have to guess at random. What is the probability you get an 80% or better on the quiz?

(**A**) $\boxed{\frac{1}{64}}$ (**B**) $\frac{1}{256}$ (**C**) $\frac{15}{1024}$ (**D**) $\frac{3}{1024}$ (**E**) None of these

6. What is the shortest distance from point $(-2,3)$ to the circle given by $(x-2)^2+(y+5)^2=5$?

(**A**) 2 (**B**) 3 (**C**) $\boxed{3\sqrt{5}}$ (**D**) $4\sqrt{5}$ (**E**) None of these

7. Find the sum of the squares of all real roots of the function $f(x)=x^4e^x-4e^x-3x^2e^x$.

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) $\boxed{8}$ (**E**) None of these

8. Find the absolute value of the sum of the solutions to the equation $(4x-6)(x+3)=14$.

(**A**) $\boxed{\frac{3}{2}}$ (**B**) $\frac{5}{2}$ (**C**) 3 (**D**) 16 (**E**) None of these

9. For how many integers x in $\{1,2,3,...,99,100\}$ is x^2+x^3 equal to the square of an integer?

(**A**) 7 (**B**) 8 (**C**) $\boxed{9}$ (**D**) 10 (**E**) None of these

10. Find the sum of the squares of all solutions of the equation $\dfrac{\log(7x-12)}{\log x}=2$.

(**A**) $\frac{16}{961}$ (**B**) $\frac{144}{25}$ (**C**) $\boxed{25}$ (**D**) 29 (**E**) None of these

11. The polynomial

$$p(x)=x^7-6x^6-12x^5+200x^4-720x^3+1248x^2-1088x+384$$

has 2 as a root of multiplicity 6. Find another root of $p(x)$.

(**A**) -32 (**B**) $\boxed{-6}$ (**C**) 6 (**D**) 32 (**E**) None of these

12. For a regular n-gon, the ratio of the number of diagonals to the number of sides is $8:1$. Find n.

(**A**) 18 (**B**) $\boxed{19}$ (**C**) 20 (**D**) 21 (**E**) None of these

13. Simplify the expression $\dfrac{(1+i)^{17}}{(1-i)^{16}}$ into $a+bi$ form.

(**A**) $\boxed{1+i}$ (**B**) $1-i$ (**C**) $-1-i$ (**D**) $-1+i$ (**E**) None of these

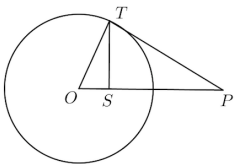
14. The point $(2,1)$ is reflected about the line $y=2x$. What are the coordinates of the resulting point?

(**A**) $(-0.5,2)$ (**B**) $(-0.3,2.1)$ (**C**) $(-0.2,2.8)$ (**D**) $\boxed{(-0.4,2.2)}$ (**E**) None of these

15. If $f(\frac{x}{2}) = x^2+x+1$, what is the largest value of z satisfying $f(3z)=13$?

(**A**) $-\dfrac{2}{3}$ (**B**) $-\dfrac{1}{2}$ (**C**) $\boxed{\dfrac{1}{2}}$ (**D**) $\dfrac{2}{3}$ (**E**) None of these

16. A circle has center O, $\overleftrightarrow{PT}$ is tangent to the circle at T, $\overline{TS} \perp \overline{OP}$, $OS=6$ and $SP=24$. Find TS.



(**A**) $\boxed{12}$ (**B**) $12\sqrt{2}$ (**C**) $6\sqrt{3}$ (**D**) $12\sqrt{3}$ (**E**) None of these

17. Find the sum of all solutions to $\sin^2 x - \sin x \cos x = \cos x - \sin x$ on $[0, 2\pi)$.

(**A**) $\dfrac{7\pi}{4}$ (**B**) $\dfrac{5\pi}{2}$ (**C**) $\boxed{3\pi}$ (**D**) $\dfrac{11\pi}{2}$ (**E**) None of these

18. How many times does the graph of the function $f(x) = \dfrac{x^3-x^2-5x-3}{x^3+4x^2-3x-18}$ cross its horizontal asymptote(s)?

(**A**) 0 (**B**) 1 (**C**) $\boxed{2}$ (**D**) 3 (**E**) None of these

19. What is the minimum value of the function $f(x) = x - 3\sqrt{x} + 9$ on its domain?

(**A**) 1.5 (**B**) 2.25 (**C**) 4.5 (**D**) $\boxed{6.75}$ (**E**) None of these

20. Find the product of the smallest solution and the largest solution of the equation

$$(x-1)(6x^2-19) + (x-4)(4x^2-11) + (6x^2-19)(x-7) = 0.$$

(**A**) $\boxed{-7}$ (**B**) 7 (**C**) $-\dfrac{49}{4}$ (**D**) $\dfrac{49}{4}$ (**E**) None of these

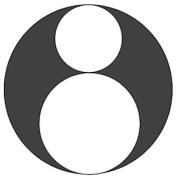
21. How many integers are in the solution set of the inequality $\dfrac{2x - \dfrac{x^2+19}{x}}{x} < 0$?

(**A**) $\boxed{8}$ (**B**) 9 (**C**) 10 (**D**) Infinitely many (**E**) None of these

22. Find the sum of the smallest and the largest x-intercepts of the graph of $y = 3x^4 - 15x^3 + 18x^2$.

(**A**) 5 (**B**) $\boxed{3}$ (**C**) 0 (**D**) -5 (**E**) None of these

23. Two circles of radii 2 and 3 are externally tangent. A third circle is circumscribed about the two smaller circles. A pin is dropped inside the largest circle. What is the probability it lands in the shaded area?



(**A**) 0.46 (**B**) $\boxed{0.48}$ (**C**) 0.50 (**D**) 0.52 (**E**) None of these

24. The perimeter of a rhombus is 10. What is the area of the rhombus?

(**A**) $\frac{30}{13}$ (**B**) $\frac{25}{4}$ (**C**) 25 (**D**) $\frac{27}{13}$ (**E**) $\boxed{\text{Not enough information}}$

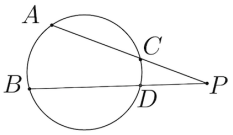
25. If the measures of all but one of the angles of a convex polygon sum to 680 degrees, what is the measure of the remaining angle in degrees?

(**A**) 20 (**B**) $\boxed{40}$ (**C**) 70 (**D**) 140 (**E**) None of these

26. The equation $|2x-1||x+5|=6$ has how many solutions which are less than zero?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) $\boxed{3}$ (**E**) None of these

27. In the figure shown, C is on $\overline{AP}$, D is on $\overline{PB}$, $PC=3$, $AC=5$ and $BD=10$. Find PD.



(**A**) 1.5 (**B**) $\boxed{2}$ (**C**) 3 (**D**) 6 (**E**) None of these

28. Find the product of the solutions of the equation $\left(\sqrt{4-\sqrt{15}}\right)^x+\left(\sqrt{4+\sqrt{15}}\right)^x=8$.

(**A**) -6 (**B**) $\boxed{-4}$ (**C**) 1 (**D**) 2 (**E**) None of these

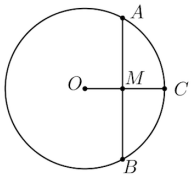
29. A projectile is launched straight up from ground level, and its height s in feet, after t seconds, can be modeled by the equation $s=-16t^2+288t$. For how long is the projectile at or above a height of 1152 ft?

(**A**) $\boxed{6 \text{ seconds}}$ (**B**) 9 seconds (**C**) 12 seconds (**D**) 18 seconds (**E**) None of these

30. Solve the equation $4\log_2 x = 4\log_4 x - 1$.

(**A**) -1 (**B**) $-\frac{1}{2}$ (**C**) $\boxed{\frac{\sqrt{2}}{2}}$ (**D**) 2 (**E**) None of these

31. In the circle centered at point O, chord $\overline{AB}$ is the perpendicular bisector of radius $\overline{OC}$. If $AB=8\sqrt{3}$, what is the area of the circle?



(**A**) 36π (**B**) 48π (**C**) $\boxed{64\pi}$ (**D**) 96π (**E**) None of these

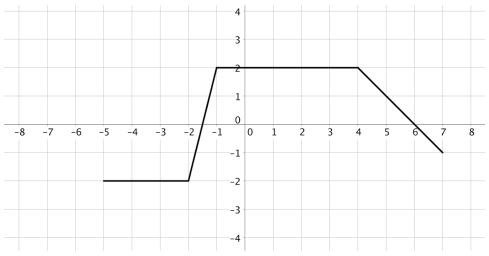
32. For two positive numbers a and b, the sum $a+b$, the product $a\cdot b$, and the difference of squares a^2-b^2 equal the same non-zero number. What is a^2-b^2?

(**A**) 2 (**B**) $\boxed{2+\sqrt{5}}$ (**C**) $\frac{3}{4}+\frac{\sqrt{5}}{2}$ (**D**) $\frac{3}{4}$ (**E**) None of these

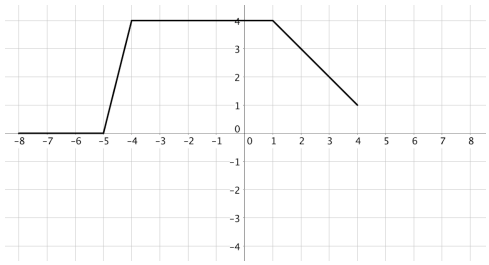
33. An after-dinner speaker anticipates delivering 35 speeches during the next 2 years. So as not to become bored, he decides to tell exactly 3 jokes in every speech, and in no two speeches to tell exactly the same 3 jokes. What is the minimum number of jokes that will accomplish this?

(**A**) $\boxed{7}$ (**B**) 37 (**C**) 70 (**D**) 105 (**E**) None of these

34. The graphs of $f(x)$ and $g(x)$ are below. Express $g(x)$ in terms of $f(x)$.



Graph of $f(x)$



Graph of $g(x)$

(**A**) $g(x) = f(x+2)+3$ (**B**) $\boxed{g(x) = f(x+3)+2}$

(**C**) $g(x) = f(x-3)+2$ (**D**) $g(x) = f(x-2)+3$ (**E**) None of these

35. Consider trapezoid $ABCD$ with $\overleftrightarrow{DC}$ parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ perpendicular to $\overleftrightarrow{AB}$. Let $DC = 2$, $DA = 8$ and $AB = 20$. If P is a point on $\overline{AB}$ so that the area of quadrilateral $APCD$ equals the area of triangle $\triangle CPB$, then find PB.

(**A**) 9 (**B**) 10 (**C**) $\boxed{11}$ (**D**) 12 (**E**) None of these

36. Find the rational value of $\log_8\left(4\sqrt{4\sqrt{4\sqrt{4\cdots}}}\right)$.

(**A**) $-\frac{1}{2}$ (**B**) $\frac{2}{3}$ (**C**) $\boxed{\frac{4}{3}}$ (**D**) 2 (**E**) None of these

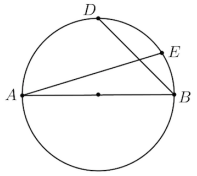
37. If $x = 2\sin\theta$, which of the following is an algebraic expression for $\cot\theta$?

(**A**) $\frac{2}{\sqrt{4-x^2}}$ (**B**) $\frac{\sqrt{4-x^2}}{2}$ (**C**) $\frac{2}{x}$ (**D**) $\frac{x}{\sqrt{4-x^2}}$ (**E**) $\boxed{\frac{\sqrt{4-x^2}}{x}}$

38. Let $f(x) = 2x^2 - 5x - 3$ and $g(x) = x^{3/2} - 4\sqrt{x}$. Find the sum of all the zeros of $(g \circ f)(x)$.

(**A**) 2.5 (**B**) $\boxed{5}$ (**C**) 6.5 (**D**) 9 (**E**) None of these

39. In the figure shown, $\overline{AB}$ is a diameter of the circle. If $m\angle EAB = 20°$, and $m\angle ABD = 50°$, determine the measure of the arc $\overset{\frown}{DE}$.



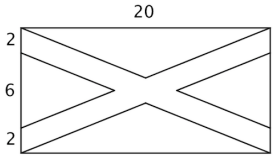
(**A**) $\boxed{40°}$ (**B**) 60° (**C**) 90° (**D**) 110° (**E**) None of these

40. Let $A = (0,0)$, $B = (1,4)$, $C = (3,-0.75)$, $D = (0,3)$, and $E = (4,0)$ be the vertices of triangles $\triangle ABC$ and $\triangle ADE$. What is the relationship between $\triangle ABC$ and $\triangle ADE$?

(**A**) $\triangle ABC \cong \triangle ADE$ (**B**) $\triangle ABC \cong \triangle AED$

(**C**) $\triangle ABC \sim \triangle ADE$ (**D**) $\boxed{\triangle ABC \sim \triangle AED}$ (**E**) None of these

41. Amelia's front yard, which measures 20 feet wide by 10 feet long, is to have two diagonal concrete sidewalks put in, as shown below. If the sidewalks measure 2 feet along the side of the yard, how much of Amelia's yard will not be concrete?



(**A**) 50 ft^2 (**B**) $\boxed{125 \text{ ft}^2}$ (**C**) $200 - 8\sqrt{29}$ ft^2

(**D**) $200 - 20\sqrt{5}$ ft^2 (**E**) None of these

42. A regular square pyramid has height $\sqrt{15}$ with the area of a triangular face equal to 10. Find the surface area of the pyramid.

(**A**) $40 + \sqrt{15}$ (**B**) $40 + 2\sqrt{15}$ (**C**) $\boxed{60}$ (**D**) 120 (**E**) None of these

43. Suppose the parabola $y = ax^2 + bx + c$ passes through the points $(-4, 12)$, $(-2, 0)$ and $(2, 12)$. Find $a + b + c$.

(**A**) $\frac{3}{4}$ (**B**) $\boxed{\frac{9}{2}}$ (**C**) $\frac{21}{2}$ (**D**) $\frac{45}{4}$ (**E**) None of these

44. The polygon(s) formed by $y = 3x + 2$, $y = -3x + 2$ and $y = -2$ is (are)

(**A**) an equilateral triangle (**B**) $\boxed{\text{an isosceles triangle}}$

(**C**) a right triangle (**D**) a triangle and a trapezoid (**E**) a quadrilateral

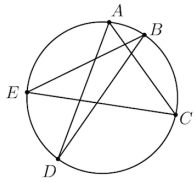
45. Find the number of distinct real values of x which have the property that the median of the five numbers $x, 6, 4, 1, 9$ is equal to their mean.

(**A**) 1 (**B**) 2 (**C**) $\boxed{3}$ (**D**) 4 (**E**) None of these

46. Find the sum of the solutions to the equation $\sin x \cos 2x + \cos x \sin 2x = 0$ in the interval $[0, 2\pi)$.

(**A**) 2π (**B**) 3π (**C**) 4π (**D**) $\boxed{5\pi}$ (**E**) None of these

47. Points A, B, C, D and E are drawn on a circle and connected to form a five pointed star. Find the value of $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$.



(**A**) 120° (**B**) $\boxed{180°}$ (**C**) 240° (**D**) 360° (**E**) None of these

48. What is the slope of the line which connects the center of the circle $(x - 4)^2 + (y + 1)^2 = 9$ and the vertex of the parabola $y = 3x^2 - 6x + 5$?

(**A**) $-\frac{1}{3}$ (**B**) $\boxed{-1}$ (**C**) $\frac{3}{5}$ (**D**) $\frac{5}{3}$ (**E**) None of these

49. How many ordered pairs (x, y), where x and y are both integers, satisfy the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$?

(**A**) 1 (**B**) 3 (**C**) 5 (**D**) $\boxed{9}$ (**E**) None of these

50. An isosceles triangle has two sides of length 5. Find the product of all possible third sides which give an area of $2\sqrt{6}$.

(**A**) $2\sqrt{6}$ (**B**) $\boxed{8\sqrt{6}}$ (**C**) 24 (**D**) 192 (**E**) None of these