Alabama Statewide Math Contest - Round 4 Division II

University of Alabama at Birmingham

April 14, 2018

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$, and $3 - 2\sqrt{3}$.

Round 4

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

Find the image of the point $(3, 2)$ after a dilation with scale factor 2 and center $(0, 1)$.

Geometry Question # 1

Answer:

Geometry Question # 1

Answer: $(6, 3)$

Geometry Question # 2

Geometry Question # 2

RESET

Quadrilateral $\square ABCD$ is inscribed in a circle such that $\overline{BD}$ is a diameter, the measure of arc $\overset{\frown}{BC}$ is $38°$, and $\triangle ABD$ is isosceles. Find the measure in degrees of angle $\angle ABC$.

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 116°

Round 4

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

A quadratic function $f(x) = ax^2 + bx + c$ has roots $x = \frac{3}{10}$, $x = -\frac{5}{6}$. If a, b, and c are all integers, what is the smallest possible value of $|a|$?

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: 60

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Let $f(x) = \dfrac{3x-2}{x+1}$. Find $f^{-1}(2)$.

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 4

Round 4

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET

The following system has one solution (a, b). Find $a + b$.

$$\begin{cases} \dfrac{x+y-1}{2x-y+2} = \dfrac{1}{3} \\ \dfrac{2x+y-2}{x+y} = \dfrac{1}{2} \end{cases}$$

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 2

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET

A circle is inscribed in a square with a diagonal of length 5. A point is chosen at random inside the square. Find the probability the point is inside the circle.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{\pi}{4}$

Round 4

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

Find the exact value of $\sin\left(\cos^{-1}\left(-\frac{4}{7}\right)\right)$

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: $\frac{\sqrt{33}}{7}$

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

A poll of fifty people found that 21 people enjoy Burger King, 13 enjoy both Burger King and McDonalds, and 23 do not like McDonalds. How many people like McDonalds but not Burger King?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 14

Round 4

Team

Team Question # 9

Team Question # 9

RESET

Find the sum of the numbers between 50 and 250 which are divisible by 8.

Team Question # 9

Answer:

Team Question # 9

Answer: 3800

Team Question # 10

Team Question # 10

RESET

Convert 206 to binary.

Team Question # 10

Answer:

Team Question # 10

Answer: 11001110_2

End of Round 4