Alabama Statewide Math Contest - Round 3 Division II

University of Alabama at Birmingham

April 14, 2018

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$, and $3 - 2\sqrt{3}$.

Round 3

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

A rhombus has sides of length 10, and its diagonals differ by 4. What is its area?

Geometry Question # 1

Answer:

Geometry Question # 1

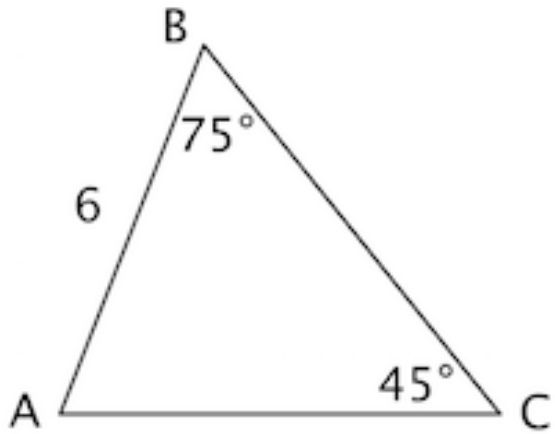
Answer: 96

Geometry Question # 2

Geometry Question # 2

RESET

In the figure below, $AB = 6$, $m(\angle ABC) = 75°$ and $m(\angle ACB) = 45°$. Find the length of side $\overline{BC}$.



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $3\sqrt{6}$

Round 3

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

Find the median of the solution set to

$$2x^3 - 5x^2 - 4x + 3 = 0$$

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: $\frac{1}{2}$

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Find the x-intercept of the line which passes through the point $(1, 2)$ and is perpendicular to the line $3x + 2y = 5$.

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: -2

Round 3

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET

A class has an average of 8.5 on a quiz. One more person takes it, receives a 7, and the average drops to 8.25. How many people originally took the quiz?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 5

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

The product of the roots of a quadratic function is -4 and their sum is $-\frac{15}{2}$. Find the absolute value of the difference of the roots.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{17}{2}$

Round 3

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

If $\log_b(x) = 2$, find $\log_b\left(\frac{1}{x^5}\right)$.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: -10

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

A fair coin is flipped five times. What is the probability that it shows exactly three heads?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{5}{16}$

Round 3

Team

Team Question # 9

Team Question # 9

RESET		

In a certain basketball-type game, the only ways to score points are with a three point basket or a five point basket. What is the largest score that is **not** attainable?

Team Question # 9

Answer:

Team Question # 9

Answer: 7

Team Question # 10

Team Question # 10

RESET		

The function $f(x) = x^3 - 8$ has three unique roots. Find the area of the triangle formed by these three roots in the complex plane.

Team Question # 10

Answer:

Team Question # 10

Answer: $3\sqrt{3}$

End of Round 3