Alabama Statewide Math Contest - Round 2 Division II

University of Alabama at Birmingham

April 14, 2018

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$, and $3 - 2\sqrt{3}$.

Round 2

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

If the two non-equal sides of a non-square rectangle are increased 3 feet each, the area will be increased 48 square feet. Find the perimeter of the original rectangle.

Geometry Question # 1

Answer:

Geometry Question # 1

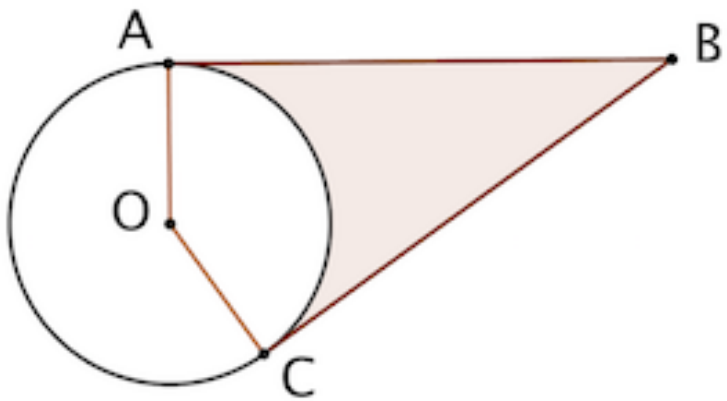
Answer: 26

Geometry Question # 2

Geometry Question # 2

RESET

In the figure below, O is the center of the circle, $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are tangent to the circle at points A and C, respectively. If the radius of the circle is 3, and $AB = 3\sqrt{3}$, find the area of the shaded region.



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $9\sqrt{3} - 3\pi$

Round 2

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

What is the remainder when $f(x) = 4x^2 + 2x + 6$ is divided by $2x - 1$?

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: 8

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Find the largest solution to $g(x) = 4$ if

$$g(x) = \begin{cases} |x| + 2 & \text{if } x \leq 0 \\ 3 + x & \text{if } 0 < x < 3 \\ x^2 & \text{if } 3 \leq x \end{cases}$$

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 1

Round 2

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET		

Find the value of $64^{1/3} + \log_{25}(5) - i^4 + \sin\dfrac{\pi}{6}$

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 4

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET

There are five marbles in a jar: two red and three black. You draw two marbles, without replacement. What is the probability that both are the same color?

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{2}{5}$

Round 2

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

The difference between two numbers is 2. Their product is 84 greater than the square of the smaller number. Find the sum of the numbers.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 86

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

Find the coefficient on the x^3 term of $(x-2)^6$.

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: -160

Round 2

Team

Team Question # 9

Team Question # 9

RESET

A rhombus $ABCD$ has $AC = 16$ and $BD = 10$. Find the $\sin(\angle BAD)$.

Team Question # 9

Answer:

Team Question # 9

Answer: $\frac{80}{89}$

Team Question # 10

Team Question # 10

RESET

A bag contains 15 gold marbles, 15 purple marbles, and 15 gray marbles. What is the minimum number of marbles you have to choose randomly from the bag to guarantee that you get 4 marbles of the same color?

Team Question # 10

Answer:

Team Question # 10

Answer: 10

End of Round 2