Alabama Statewide Math Contest - Round 1 Division II

University of Alabama at Birmingham

April 14, 2018

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$,and $3 - 2\sqrt{3}$.

Round 1

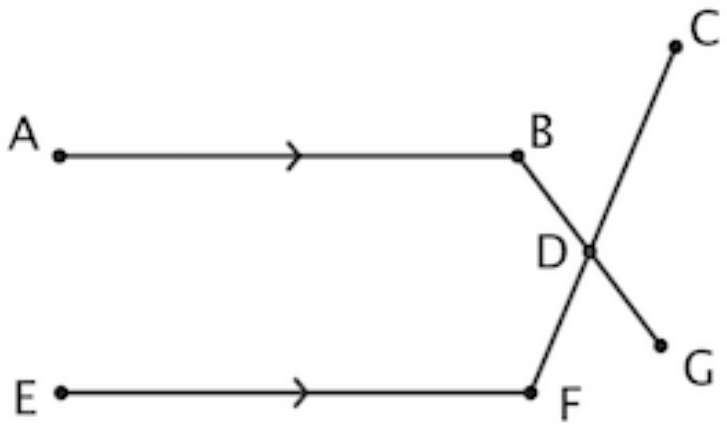
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

In the figure below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$, D is on $\overline{CF}$ and $\overline{BG}$, $m(\angle EFD) = 106°$, and $m(\angle CDG) = 150°$. Find $m(\angle ABD)$ in degrees.



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 104°

Geometry Question # 2

Geometry Question # 2

RESET

Find the surface area of a right prism with regular hexagon bases of side length 2, and a height of 3.

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $36 + 12\sqrt{3}$

Round 1

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

Let $f(x) = x - \frac{1}{x}$, and $g(x) = x^2$. Find the smallest real solution to the equation $(f \circ g)(x) = 0$.

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: -1

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Define an operation $\bigstar$ as $(x, y)\bigstar(z, w) = xz^2 - yw$ for x, y, z, w real numbers. Find $(6, 2)\bigstar(-1, 4)$.

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: -2

Round 1

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET		

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & -1 \end{bmatrix}$, find $a + b + c + d$.

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 9

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Solve the equation $\log\left(x^2\right) - \log\left(5x\right) = \log\left(6\right)$.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: 30

Round 1

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

Find $a + b$ if $a + bi = \dfrac{(5 - 2i)^2}{1 + i}$.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: -20

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

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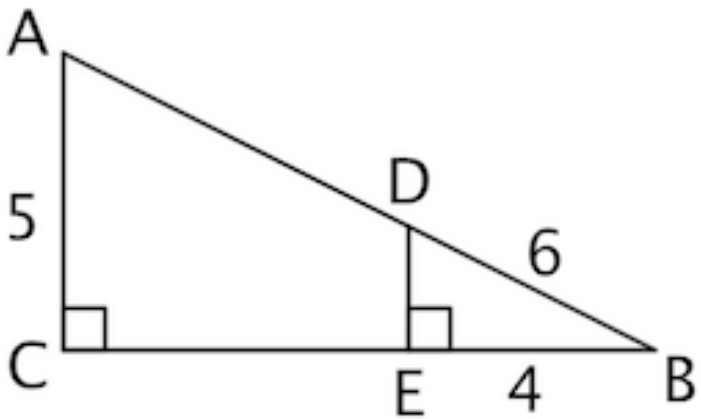
In the figure below, D is on $\overline{AB}$, E is on $\overline{BC}$, $AC = 5$, $BD = 6$, and $BE = 4$, where $\angle ACE$ and $\angle DEB$ are right angles. Find $\cos(\angle DAC)$.



Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{\sqrt{5}}{3}$

Round 1

Team

Team Question # 9

Team Question # 9

RESET

Let A be the largest two digit prime number whose digits are also prime, B be the degree measure of an interior angle in a regular pentagon, and C be the sixth Fibonacci number. Find $A + B + C$.

Team Question # 9

Answer:

Team Question # 9

Answer: 189

Team Question # 10

Team Question # 10

RESET

Let x and y be real numbers. What is the largest value of x for which (x, y) satisfies $x^2 + y^2 = 8x - 6y - 16$.

Team Question # 10

Answer:

Team Question # 10

Answer: 7

End of Round 1