Alabama Statewide Math Contest - Round 1 Division 2

University of North Alabama

April 8, 2017

Scoring

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

RESET		

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$,and $3 - 2\sqrt{3}$.

Round 1

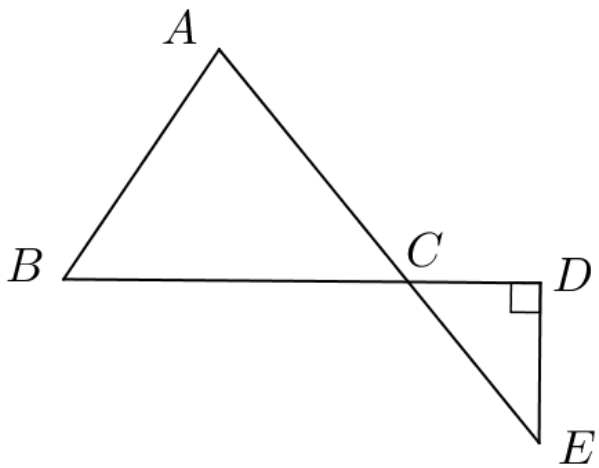
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

In the given figure, triangle $\triangle ABC$ is equilateral, $\overline{AE}$ and $\overline{BD}$ intersect at point C, and $\angle CDE$ is a right angle. If $AB = 8$ and $AE = 20$, what is BD?



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 14

Geometry Question # 2

Geometry Question # 2

RESET

A parallelogram is given by the points $(0, 0)$, $(1, 2)$, $(7, 2)$, and $(6, 0)$ What is the point at which its diagonals cross?

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $\left(\frac{7}{2}, 1\right)$

Round 1

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

What is the x-intercept of the line which is perpendicular to $3x + 2y = 5$, and which has the same y-intercept as the line $4x - 3y = 8$?

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: 4 or $(4, 0)$

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

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Alyce has been married for 10 years. She calculates that in 20 years, she will have been married for three years more than half her life. At what age did Alyce get married?

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 24

Round 1

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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The parabola given by $y = x^2 + 6x - 11$ and the line given by $y = 3x - 7$ intersect twice. Find the distance between these intersection points.

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: $5\sqrt{10}$

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

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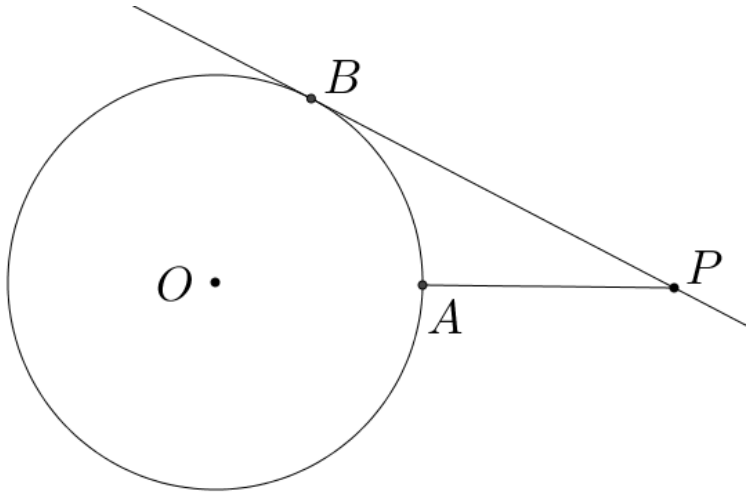
A point P has a distance of 7 to a circle of radius 3. What is the distance from P to the circle along the tangent $\overline{PB}$?



Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\sqrt{91}$

Round 1

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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The value $x = 2$ is one solution to the equation

$$x^3 - 9x^2 + 18x - 8 = 0$$

Find the sum of the other two solutions.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 7

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

Let N denote a six-digit integer whose 6 digits are 1, 2, 3, 4, 5, 6 in a random order. What is the probability that N is divisible by 6?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{1}{2}$

Round 1

Team

Team Question # 9

Team Question # 9

RESET		

What is the number of non-congruent rectangles with integer side lengths and area 220?

Team Question # 9

Answer:

Team Question # 9

Answer: 6

Team Question # 10

Team Question # 10

RESET		

Find the sum of all zeros of the piecewise defined function

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \leq 0 \\ 4x - 5 & \text{for } x > 0 \end{cases}$$

Team Question # 10

Answer:

Team Question # 10

Answer: $\frac{1}{4}$

End of Round 1