The 35th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 27, 2016 at Regional Testing Centers
Second Round: April 9, 2016 at The University of North Alabama

ALGEBRA II WITH TRIGONOMETRY EXAM

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. Find the slope of the line between the points $(\sqrt{3}, \sqrt{6})$ and $(2, 4\sqrt{2})$.

(**A**) $6\sqrt{6} - 11\sqrt{2}$ (**B**) $\dfrac{5\sqrt{2} + 2\sqrt{6}}{26}$ (**C**) $5\sqrt{2}$ (**D**) $\boxed{5\sqrt{2} + 2\sqrt{6}}$ (**E**) None of these

2. How many values of x satisfy the radical equation $\sqrt{x+2} + \sqrt{3x+7} = 1$?

(**A**) 0 (**B**) $\boxed{1}$ (**C**) 2 (**D**) Infinitely Many (**E**) None of these

3. In last year's NBA Finals, LeBron James scored a total of 169 points from 2-pointers and 3-pointers. If he made a total of 78 shots, how many 3-pointers did he make?

(**A**) 65 (**B**) 45 (**C**) 25 (**D**) 15 (**E**) $\boxed{\text{None of these (13)}}$

4. Simplify the expression

$$\frac{3^5 \cdot (27^2)^3 \cdot \sqrt{3}}{81 \cdot 9^{5/4}}$$

(**A**) $3^{14.5}$ (**B**) 3^{16} (**C**) $\boxed{3^{17}}$ (**D**) $3^{18.25}$ (**E**) None of these

5. The equation $(x^2 - x - 4)^{3/4} - 2 = 6$ has two solutions, a and b. Find $a^2 + b^2$.

(**A**) 1 (**B**) 9 (**C**) 25 (**D**) $\boxed{41}$ (**E**) None of these

6. Consider the polynomial $y = x^2 - 6x + 11$. Find the maximum $y-$value that this graph obtains on the interval $[0, 5]$.

(**A**) 2 (**B**) 3 (**C**) $\boxed{11}$ (**D**) 18 (**E**) None of these

7. An arithmetic sequence has a 7th term of 5 and a 12th term of 20. Which term is equal to the sum of the first 10 terms?

(**A**) 5th (**B**) $\boxed{\text{7th}}$ (**C**) 10th (**D**) 14th (**E**) None of these

8. Find the absolute value of the sum of the solutions to the equation $(3x - 2)(x + 3) = 14$.

(**A**) $\boxed{\frac{7}{3}}$ (**B**) $\frac{10}{3}$ (**C**) $\frac{11}{3}$ (**D**) $\frac{49}{3}$ (**E**) None of these

9. The College World Series for baseball is decided by a best of three game series; that is, the first team to win two games is the winner. If Texas and Florida are playing in the championship, and for any given game Texas has a 60% chance of beating Florida, what is the probability that the series will need to play the third game?

(**A**) 0.24 (**B**) 0.36 (**C**) $\boxed{0.48}$ (**D**) 0.52 (**E**) None of these

10. If $x + y = 3$ and $x^2 + y^2 = 6$, find $x^3 + y^3$.

(**A**) 7.5 (**B**) 9 (**C**) $\boxed{13.5}$ (**D**) 27 (**E**) None of these

11. Find the sum of the absolute values of all numbers x such that $f(x)$ is undefined, where

$$f(x) = \frac{2x^2 + 11x - 6}{3x^3 + 5x^2 - 12x - 20}.$$

(**A**) $\boxed{\frac{17}{3}}$ (**B**) $\frac{13}{2}$ (**C**) $\frac{23}{5}$ (**D**) $\frac{11}{3}$ (**E**) None of these

12. Find the distance between the foci of the ellipse given by the equation $\frac{(x-3)^2}{7} + \frac{(y+1)^2}{2} = 1$.

(**A**) 3 (**B**) $\boxed{2\sqrt{5}}$ (**C**) 6 (**D**) $6\sqrt{5}$ (**E**) None of these

13. On a date night menu, you and your date get to choose one appetizer to share from a list of three, two entrees from a list of five, and one dessert to share from a list of four. Assuming you both could order the same entree, how many different meals are possible?

(**A**) 120 (**B**) 160 (**C**) 240 (**D**) $\boxed{300}$ (**E**) None of these

14. The system of equations listed below has two solutions (a, b) and (c, d), where $a < c$. Find the number $a - b + 2c + 3d$.

$$\begin{cases} \frac{1}{x} + \frac{1}{y} &= \frac{1}{2} \\ \frac{2}{xy} &= \frac{1}{9} \end{cases}$$

(**A**) -27 (**B**) $\boxed{18}$ (**C**) 27 (**D**) 30 (**E**) None of these

15. Define the function $f(x-2) = (x-2)^2 - 3(x-3) + 1$. Find $f(-1)$.

(**A**) -2 (**B**) 5 (**C**) $\boxed{8}$ (**D**) 22 (**E**) None of these

16. Find the horizontal asymptote of the graph of the function $f(x) = \frac{3^{x+2}(x+1)}{3^x x}$.

(**A**) $y = 0$ (**B**) $y = 3$ (**C**) $y = 6$ (**D**) $\boxed{y = 9}$ (**E**) $y = 12$

17. Find the sum of all solutions to the equation $|x+3|^2 + 7|x+3| - 18 = 0$.

(**A**) $\boxed{-6}$ (**B**) -7 (**C**) -12 (**D**) 7 (**E**) None of these

18. On a standard clock, what is the angle formed, in degrees, between the hands when it is 4:30?

(**A**) $\boxed{45°}$ (**B**) $50°$ (**C**) $55°$ (**D**) $60°$ (**E**) None of these

19. If $f(x) = \frac{1}{\sqrt{16 - x^2}}$, how many integers are in the domain of $f(x)$?

(**A**) 2 (**B**) $\boxed{7}$ (**C**) 9 (**D**) Infinitely many (**E**) None of these

20. Define two new operations on real numbers as follows

$$a \bowtie b = a^b \qquad a \circledast b = 2a + b$$

Calculate the value of $(16 \bowtie \frac{1}{2}) \circledast (2 \bowtie 3)$.

(**A**) 14 (**B**) $\boxed{16}$ (**C**) 22 (**D**) 24 (**E**) None of these

21. Find the value of $i \cdot i^2 \cdots i^{63}$, where $i = \sqrt{-1}$.

(**A**) $\boxed{1}$ (**B**) -1 (**C**) i (**D**) $-i$ (**E**) None of these

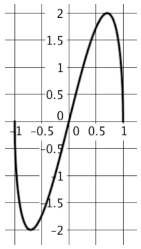
22. Jim begins with only \$1 on day one and doubles his money daily. On day 24, he reaches his goal of x dollars. Assuming the same earnings rate, on which day would Jim reach the same goal if he started with \$8?

(**A**) 13 (**B**) 14 (**C**) 20 (**D**) $\boxed{21}$ (**E**) None of these

23. If $f^{-1}(x) = \sqrt{x-3}+5$, is the inverse of the function $f(x)$, find the solution to the equation $f(x) = 52$.

(**A**) -2 (**B**) $\boxed{12}$ (**C**) 212 (**D**) 2212 (**E**) None of these

24. Which of the following characteristics does the function graphed shown possess?



I. Even II. Odd III. Domain $[-2, 2]$ IV. Range $[-2, 2]$

(**A**) I and III (**B**) I and IV (**C**) II and III (**D**) $\boxed{\text{II and IV}}$ (**E**) III and IV

25. Find the radius of the circle whose equation is $x^2 + y^2 - 16x - 10y + 64 = 0$.

(**A**) $\boxed{5}$ (**B**) 8 (**C**) 25 (**D**) 64 (**E**) None of these

26. At a game of trivia, you are asked to match four events to four different years in which they happened. If you randomly guess the answer, what is the probability that you guessed correctly?

(**A**) $\frac{1}{256}$ (**B**) $\boxed{\frac{1}{24}}$ (**C**) $\frac{1}{10}$ (**D**) $\frac{1}{4}$ (**E**) None of these

27. Let $x = \frac{1}{2}$ radians. Find the value of $\log_3(\tan x) - \log_3(\sin x) + \log_3(\cos x)$.

(**A**) $\boxed{0}$ (**B**) 1 (**C**) 3 (**D**) Undefined (**E**) None of these

28. For which of the following values of k does the equation $\dfrac{x-1}{x-2} = \dfrac{x-k}{x-6}$ have no solution for x?

(**A**) 1 (**B**) 3 (**C**) $\boxed{5}$ (**D**) 7 (**E**) None of these

29. Define an operator mod on two integers a and b as follows: a mod $b = q$ if the q is the remainder of a divided by b. Calculate 2016 mod 5.

(**A**) 0 (**B**) $\boxed{1}$ (**C**) 2 (**D**) 3 (**E**) None of these

30. Find the exact value of $\dfrac{(10!)^2 - (9!)^2}{(10!)^2 + (9!)^2}$.

(**A**) $\frac{9}{11}$ (**B**) $\boxed{\frac{99}{101}}$ (**C**) $\frac{81}{121}$ (**D**) $\frac{19}{181}$ (**E**) None of these

31. If $\dfrac{7x^2 - 13x + 13}{(x-2)(x^2-2x+3)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2-2x+3}$, find the product ABC.

(**A**) $\boxed{10}$ (**B**) 17 (**C**) 29 (**D**) 45 (**E**) None of these

32. Find the number of distinct solutions to the equation $\sqrt[3]{5x} = \sqrt{2x}$.

(**A**) 0 (**B**) 1 (**C**) $\boxed{2}$ (**D**) 3 (**E**) None of these

33. The line perpendicular to $4x + 2y = 5$, and with the same y-intercept, is graphed on the coordinate plane. What is the sum of the non-zero coordinates of its x- and y-intercepts?

(**A**) $\boxed{-2.5}$ (**B**) 1.25 (**C**) 3.75 (**D**) 7.5 (**E**) None of these

34. Find the inverse function $f^{-1}(x)$ of the function $f(x) = x^2 - 4x + 3$ if $x \leq 2$.

(**A**) $f^{-1}(x) = \dfrac{1}{x^2 - 4x + 3}$ (**B**) $f^{-1}(x) = -\sqrt{x-3} + 2$

(**C**) $\boxed{f^{-1}(x) = -\sqrt{x+1} + 2}$ (**D**) $f^{-1}(x) = -\sqrt{x+3}$ (**E**) $f^{-1}(x) = -x^2 + 4x - 3$

35. In a bin, there are 15 unique marbles: 4 white, 9 purple and 2 gold. You choose a handful of three marbles. In how many ways can we choose the three marbles so that exactly two of them are white?

(**A**) 6 (**B**) $\boxed{66}$ (**C**) 90 (**D**) 132 (**E**) None of these

36. The function $f(x) = 2x^3 - 15x^2 + 19x - 6$ has a root at $x = 1$. Find the product of the squares of the other two roots.

(**A**) $\boxed{9}$ (**B**) 18 (**C**) 81 (**D**) 324 (**E**) None of these

37. Clara is on a bike ride. If she bikes uphill 4 miles at a rate of 8 miles per hour and then bikes downhill 4 miles at a rate of 16 miles per hour, what was her average speed on the trip, rounded to the nearest integer?

(**A**) 10 (**B**) $\boxed{11}$ (**C**) 12 (**D**) 13 (**E**) None of these

38. Find the sum of the squares of all values of x so that $f(x) = 8$ if

$$f(x) = \begin{cases} x^2 + 2x & x < 0 \\ 3x - 7 & x \geq 0 \end{cases}$$

(**A**) 20 (**B**) 25 (**C**) $\boxed{41}$ (**D**) 45 (**E**) None of these

39. If $f(x) = a + bx$, there are two such pairs (a, b) such that $f(f(1)) = \frac{23}{2}$ and $f(f(0)) = \frac{5}{2}$. What is the maximum value of $\frac{a}{b}$?

(**A**) $\frac{5}{24}$ (**B**) $\frac{5}{18}$ (**C**) $\boxed{\frac{5}{12}}$ (**D**) $\frac{5}{6}$ (**E**) None of these

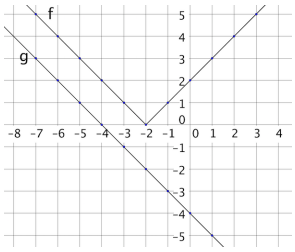
40. Suppose that $f(x) = x^x$ and $g(x) = x^{2x}$. Which of the functions below is equivalent to $(f \circ g)(x)$?

(**A**) x^{3x} (**B**) $x^{x^{2x}}$ (**C**) $x^{2x^{x+1}}$ (**D**) $\boxed{x^{2x^{2x+1}}}$ (**E**) $(2x)^{2x}$

41. Given that $\log_{10} 2 = x$, and $\log_{10} 3 = y$, what is $\log_{10} 12$?

(**A**) $2xy$ (**B**) $\boxed{2x + y}$ (**C**) $x^2 + y$ (**D**) $x^2 y$ (**E**) None of these

42. Find $f(g(1))$ given the graphs of f and g shown.



(**A**) 1 (**B**) $\boxed{3}$ (**C**) -15 (**D**) Undefined (**E**) None of these

43. The difference quotient of a function $f(x)$ is the quotient $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$. Find the difference quotient of x^2+1.

(**A**) 1 (**B**) h (**C**) $\dfrac{h^2+1}{h}$ (**D**) $2x$ (**E**) $\boxed{2x+h}$

44. Find the value of c so that $x=1$ is not a vertical asymptote of the graph of the function

$$y = \frac{x^2-4x+c}{x^2+x-2}.$$

(**A**) -12 (**B**) -5 (**C**) $\boxed{3}$ (**D**) 4 (**E**) None of these

45. The function $k(x) = \sqrt{\sin(5x)}$ is the composition of the functions $f(x) = \sin x$, $g(x) = \sqrt{x}$ and $h(x) = 5x$. Which of the following represents the correct order of composition?

(**A**) $k(x) = (f \circ g \circ h)(x)$ (**B**) $\boxed{k(x) = (g \circ f \circ h)(x)}$

(**C**) $k(x) = (f \circ h \circ g)(x)$ (**D**) $k(x) = (h \circ g \circ f)(x)$ (**E**) None of these

46. Find the sum of all solutions to the absolute value equation $|7x+9| = x^2+1$.

(**A**) -14 (**B**) -7 (**C**) $\boxed{0}$ (**D**) 7 (**E**) None of these

47. The function $p(x) = -x^2+46x-360$ models the daily profits, in hundreds of dollars, for a small company that produces x bicycles per day. How many bicycles should be made each day to maximize profit?

(**A**) $\boxed{23}$ (**B**) 46 (**C**) 169 (**D**) 180 (**E**) None of these

48. Find the sum of all solutions to the equation $2x^{2/3} - x^{1/3} = 6$.

(**A**) $-\frac{37}{8}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{91}{8}$ (**D**) $\boxed{\frac{37}{8}}$ (**E**) None of these

49. For how many values of k does $x^3+x^2-4kx-4k=0$ has two of its three roots equal?

(**A**) 0 (**B**) 1 (**C**) $\boxed{2}$ (**D**) 3 (**E**) None of these

50. Form a 7-digit number, n, at random using the digits 1 through 7 exactly once. What is the probability that n is divisible by 25?

(**A**) $\frac{1}{14}$ (**B**) $\boxed{\frac{1}{21}}$ (**C**) $\frac{1}{42}$ (**D**) $\frac{1}{2520}$ (**E**) None of these