The 34th Annual

ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round: February 28, 2015 at Regional Testing Centers
Second Round: April 11, 2015 at University of North Alabama

ALGEBRA II WITH TRIGONOMETRY EXAM

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

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Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer
- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining
- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

The following information is courtesy of the U.S. Bureau of Labor Statistics.

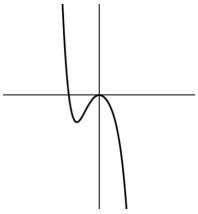
- The median salary of a Mathematician in 2012 was $101,360 per year.
- Over the next 10 years, the job opportunities for mathematicians are expected to grow by 23%!

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new Mathematics Fellow program, an active undergraduate research group and a new Dual Degree Engineering program. For more information, go to www.una.edu/math.

1. For $f(x) = 6x - 4(2x-1)^2 + 5$, find $f(2)$.

(**A**) -127 (**B**) -29 (**C**) $\boxed{-19}$ (**D**) 77 (**E**) None of these

2. Which of the following could be the leading term of the polynomial whose graph is as pictured?



(**A**) $3x^3$ (**B**) $-x^4$ (**C**) $\boxed{-2x^5}$ (**D**) $8x^6$ (**E**) None of these

3. What is the largest solution of the equation $2x^3 - 5x^2 = 8x - 20$?

(**A**) 2 (**B**) $\frac{7}{3}$ (**C**) 4 (**D**) $\frac{9}{2}$ (**E**) $\boxed{\text{None of these}}$

4. Which of the following functions is/are equal to $f(x) = 4^x$?

I. $g(x) = 2^{2^x}$ II. $h(x) = (2^x)^2$ III. $k(x) = 2^{2x}$

(**A**) I only (**B**) $\boxed{\text{II and III only}}$ (**C**) I and III only (**D**) I, II and III (**E**) II only

5. The number of values satisfying the equation $\dfrac{2x^2 - 10x}{x^2 - 5x} = x - 3$ is

(**A**) $\boxed{0}$ (**B**) 1 (**C**) 2 (**D**) 3 (**E**) None of these

6. The difference quotient of a function $f(x)$ is the quotient $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$. Find the difference quotient of $f(x) = \frac{3}{x}$.

(**A**) $\frac{3}{x(x+h)}$ (**B**) 1 (**C**) $\frac{1}{x(x+h)}$ (**D**) $\boxed{\frac{-3}{x(x+h)}}$ (**E**) $\frac{3}{h^2}$

7. In a warehouse, a stack of 6 foam mattress toppers are piled up. Each mattress topper is originally 3 inches thick. Each is compressed by a third each time an additional mattress topper is piled on top. Rounded to the nearest inch, which of the following is the total height of the pile?

(**A**) 4 (**B**) 6 (**C**) $\boxed{8}$ (**D**) 10 (**E**) 12

8. Find the sum of the solutions to the equation $(3x - 10)(x + 1) = 10$

(**A**) $\frac{17}{5}$ (**B**) $\boxed{\frac{7}{3}}$ (**C**) $\frac{47}{3}$ (**D**) $-\frac{13}{5}$ (**E**) None of these

9. The student council is made up of four sophomores, two juniors and three seniors. A yearbook photographer would like to line up all members of the student council in a line for a picture. How many different pictures are possible if students in the same grade stand beside each other?

(**A**) $\boxed{4!2!3!3!}$ (**B**) 3! (**C**) 4!2!3! (**D**) 9! (**E**) None of these

10. For $f(x)$ defined piecewise below, evaluate $f(f(f(f(-3))))$.

$$f(x) = \begin{cases} |x| & \text{if} \quad x < -2 \\ -5 & \text{if} \quad -2 \le x \le 3 \\ \frac{1}{x} & \text{if} \quad x > 3 \end{cases}$$

(**A**) $\boxed{\frac{1}{5}}$ (**B**) 3 (**C**) -5 (**D**) 5 (**E**) None of these

11. Find the horizontal asymptote of the graph of the function $f(x) = \dfrac{-6x^2+5x}{4x+5} + \dfrac{12x^3+7x+1}{8x^2+6x-5}$.

(**A**) $y = \frac{1}{2}$ (**B**) $y = 0$ (**C**) $\boxed{y = 2}$ (**D**) $y = -\frac{5}{4}$ (**E**) None of these

12. If $x + \dfrac{1}{x} = 3$, then the value of $x^3 + \dfrac{1}{x^3}$ is

(**A**) 9 (**B**) $\boxed{18}$ (**C**) 24 (**D**) 27 (**E**) None of these

13. Find the slope of the line perpendicular to the line connecting the two points $(\frac{1}{4}, -\frac{1}{3})$ and $(-\frac{1}{6}, 1)$.

(**A**) $-\frac{16}{5}$ (**B**) $\frac{16}{5}$ (**C**) $-\frac{5}{16}$ (**D**) $\boxed{\frac{5}{16}}$ (**E**) None of these

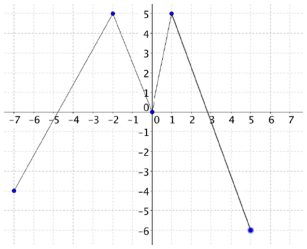
14. In the senior class at a particular high school, 45 students are taking calculus, 52 students are taking physics, and 21 students are taking both calculus and physics. If there are 200 people in the senior class, what is the probability that a randomly selected student is taking calculus or physics?

(**A**) $\boxed{0.38}$ (**B**) 0.97 (**C**) 0.485 (**D**) 0.59 (**E**) None of these

15. To which of the following expressions is $\sqrt[8]{x^8} + \sqrt[7]{x^7}$ equal to for all real negative values of x?

(**A**) x (**B**) $2x$ (**C**) $2x^2$ (**D**) $\boxed{0}$ (**E**) x^2

16. The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 5$ have?



(**A**) 2 (**B**) 3 (**C**) 4 (**D**) $\boxed{6}$ (**E**) None of these

17. Exactly how many solutions are there to the equation $|x-2| + |x-3| = 1$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) $\boxed{\text{None of these}}$

18. If $\frac{2}{5}$ and $\frac{14}{15}$ are the first and fifth terms of an arithmetic sequence, what is the sum of the second, third and fourth terms of the sequence?

(**A**) $\boxed{2}$ (**B**) $\frac{24}{5}$ (**C**) $\frac{8}{5}$ (**D**) $\frac{8}{3}$ (**E**) None of these

19. Suppose that $f(x) = x^2 - 6x$, $g(x) = x + 3$ and $h(x) = \sqrt{x}$. Find $K(x)$, where $K(x) = (f \circ (g \circ h))(x)$.

(**A**) $\boxed{K(x) = x - 9}$ (**B**) $K(x) = x - 6\sqrt{x} - 9$ (**C**) $K(x) = -5x + 9$

(**D**) $K(x) = \sqrt{x^2 - 6x + 3}$ (**E**) $K(x) = -5x + 6\sqrt{x} + 9$

20. Let f be a function satisfying $f(xy) = \dfrac{f(x)}{y}$ for all positive real numbers x and y. If $f(500) = 3$, what is the value of $f(600)$?

(**A**) $\frac{18}{5}$ (**B**) $\boxed{\frac{5}{2}}$ (**C**) 103 (**D**) $\frac{6}{5}$ (**E**) None of these

21. The graph of the function $y = \dfrac{x^2 - 5x + 6}{2x^2 + 3x - 4}$ has two vertical asymptotes $x = a$ and $x = b$. Find ab.

(**A**) -6 (**B**) -4 (**C**) $\boxed{-2}$ (**D**) 6 (**E**) None of these

22. A line with slope 3 intersects a line with slope $-\frac{1}{2}$ at the point $(14, 15)$. What is the distance between the x-intercepts of these lines?

(**A**) 5 (**B**) $\boxed{35}$ (**C**) 49 (**D**) 53 (**E**) None of these

23. Six houses in a row are each to be painted with one of the colors Red, Blue, Green and Yellow. In how many different ways can the houses be painted so that no two adjacent houses are the same color?

(**A**) $\boxed{972}$ (**B**) 720 (**C**) 288 (**D**) 1728 (**E**) None of these

24. A fixed point of a function f is a value c in the domain of f such that $f(c) = c$. Find the number of fixed points of the function $f(x) = \dfrac{1}{x + 2}$.

(**A**) 0 (**B**) 1 (**C**) $\boxed{2}$ (**D**) infinitely many (**E**) None of these

25. Which of the following best describes the graph of the equation $9x^2 - 54x = 4y^2 + 16y - 29$?

(**A**) Two straight lines (**B**) $\boxed{\text{Hyperbola}}$ (**C**) Ellipse (**D**) Circle (**E**) None of these

26. The equation $2x^{5/6} + 2x^{1/2} = 5x^{2/3}$ has two nonzero real solutions, a and b. Assuming $a < b$, find $\frac{1}{a} + b$.

(**A**) 4 (**B**) 16 (**C**) 64 (**D**) $\boxed{128}$ (**E**) None of these

27. Suppose $f(1) = 3$ and for all integers $n > 1$, $f(n) = f(n-1) + 1$ if n is even and $f(n) = 2f(n-1)$ if n is odd. Find $f(5)$.

(**A**) 10 (**B**) $\boxed{18}$ (**C**) 6 (**D**) 14 (**E**) None of these

28. Find the remainder when the polynomial $f(x) = 2x^4 + 5x^3 - 3x - 8$ is divided by the polynomial $x + 2$.

(**A**) 58 (**B**) -20 (**C**) 2 (**D**) 22 (**E**) $\boxed{\text{None of these}}$

29. On an island, 99% of the population are natives. Some natives emigrate so that only 98% of the population are natives. If the initial population of the island was 1000, how many natives emigrated?

(**A**) 10 (**B**) 20 (**C**) 100 (**D**) $\boxed{500}$ (**E**) None of these

30. Find the inverse function $f^{-1}(x)$ of the function $f(x) = \dfrac{x+2}{x+3}$.

(**A**) $f^{-1}(x) = \frac{3x-2}{x+2}$ (**B**) $f^{-1}(x) = \frac{-1}{x-1}$ (**C**) $f^{-1}(x) = \frac{x+3}{x+2}$

(**D**) $\boxed{f^{-1}(x) = \frac{2-3x}{x-1}}$ (**E**) $f^{-1}(x) = \frac{6+3x}{6+2x}$

31. Find the product of all real numbers k for which the function $f(x) = kx^2 + x + k$ touches, but does not cross, the x-axis.

(**A**) $\frac{1}{2}$ (**B**) 0 (**C**) $\boxed{-\frac{1}{4}}$ (**D**) -1 (**E**) None of these

32. The graphs of equations $9x^2 + y^2 = 9$ and $3x + 2y = 6$ intersect at points (a, b) and (c, d). Find $a + b + c + d$.

(**A**) $\boxed{\frac{28}{5}}$ (**B**) $-\frac{13}{8}$ (**C**) 4 (**D**) 0 (**E**) None of these

33. Let $f(x)$ be a function such that $f(x) + f(\frac{1}{1-x}) = x$ for all x not equal to 0 or 1. What is the value of $f(2)$?

(**A**) $\boxed{\frac{7}{4}}$ (**B**) 1 (**C**) $\frac{3}{2}$ (**D**) $\frac{3}{4}$ (**E**) None of these

34. The following transformations are applied (in the given order) to the graph of $y = |x|$.

I. Reflection about the x-axis

II. Horizontal shift left 3 units

III. Vertical shift up 1 unit

Determine the equation of the graph produced as a result of applying these transformations.

(**A**) $y = -|x + 3| - 1$ (**B**) $y = -|x + 1| - 3$

(**C**) $y = -|x - 1| + 3$ (**D**) $y = -|x - 3| + 1$ (**E**) $\boxed{\text{None of these}}$

35. Find the sum of all solutions (both real and complex) to the equation $ix^2 + 7x - 12i = 0$, where $i = \sqrt{-1}$.

(**A**) $-7i$ (**B**) $-i$ (**C**) $\boxed{7i}$ (**D**) $8i$ (**E**) None of these

36. Define an operation # on pairs of real numbers as

$$(x_1, y_1)\#(x_2, y_2) = (x_1^2 x_2^2, y_1 y_2).$$

Which of the following could **not** equal $(x_1, y_1)\#(x_2, y_2)$ for any real numbers x_1, y_1, x_2, y_2?

(**A**) $(0, \sqrt{2})$ (**B**) $\boxed{(-1, 0)}$ (**C**) $(1, 4)$ (**D**) $(\pi, -7)$ (**E**) None of these

37. The straight lines $-ax + \frac{1}{2}y = 1$ and $(a+1)x + y = 1$ are parallel to each other. Find the value of the constant a.

(**A**) $-\frac{1}{2}$ (**B**) $\frac{-1+\sqrt{3}}{2}$ (**C**) $\boxed{-\frac{1}{3}}$ (**D**) 1 (**E**) None of these

38. A box contains 1 white, 3 purple, and 2 gold balls. A second box contains 2 purple and 2 gold balls. One ball is selected at random from each box. What is the probability they are the same color?

(**A**) $\frac{5}{6}$ (**B**) $\frac{16}{45}$ (**C**) $\boxed{\frac{5}{12}}$ (**D**) $\frac{1}{2}$ (**E**) None of these

39. Let a and d be real numbers such that $ax^3 + 6x^2 + 9x + d = (f(x))^3$ for some polynomial function $f(x)$. What is the value of ad?

(**A**) $\boxed{6}$ (**B**) 1 (**C**) 54 (**D**) 8 (**E**)

40. At a Presidents' Day sale, you buy a scarf regularly priced at \$20. You also buy a sweater without a price tag which is on a 50% off rack. After using a 10% off your total purchase coupon, you pay \$45. Assuming there is no tax, what was the original price of the sweater?

(**A**) \$67.50 (**B**) $\boxed{\$60.00}$ (**C**) \$55.56 (**D**) \$50.00 (**E**) None of these

41. The parabola with the equation $y = ax^2+bx+c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2+ex+f$. Which of the following equal $a+b+c+d+e+f$?

(**A**) $2b$ (**B**) $2c$ (**C**) $2a + 2b$ (**D**) $2h$ (**E**) $\boxed{2k}$

42. Which of the following functions has a graph with exactly one hole, occurring when $x = 1$ and exactly one vertical asymptote, occurring when $x = -7$?

(**A**) $f(x) = \dfrac{x^4 - x^3}{x^3 + 6x^2 - 7x}$ (**B**) $f(x) = \dfrac{x^4 + 7x^3}{x^2 + 6x - 7}$

(**C**) $\boxed{f(x) = \dfrac{x^4 - x^3}{x^2 + 6x - 7}}$ (**D**) $f(x) = \dfrac{x^4 + 7x^3}{x^3 + 6x^2 - 7x}$ (**E**) None of these

43. Let $S = 1 + 2 + 3 + \cdots + 10^n$, for n a positive integer. How many factors of 2 are there in the prime factorization of S?

(**A**) 10^n (**B**) 0 (**C**) $\boxed{n-1}$ (**D**) $2n - 1$ (**E**) None of these

44. Amelia is driving at 60 mph, tapping her toe to a song at a rate of 5 taps per 3 seconds. She notices that every 10th tap always falls on a telephone pole. Assuming the telephone poles are always an equal distance apart on this road, how many miles apart are they?

(**A**) $\frac{1}{6}$ (**B**) $\boxed{\frac{1}{10}}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{1}{40}$ (**E**) None of these

45. Find the value of k so that the function $f(x)$ is continuous.

$$f(x) = \begin{cases} kx^2 + 3 & x < 2 \\ k & x \geq 2 \end{cases}$$

(**A**) 2 (**B**) $-\frac{3}{4}$ (**C**) $-\frac{1}{4}$ (**D**) 1 (**E**) $\boxed{\text{None of these}}$

46. Let (a, b) be a point on the graph of $16x^2 + 36y^2 - 8x + 48y = 127$. What is the largest such b?

(**A**) $\frac{13}{4}$ (**B**) $\frac{8}{3}$ (**C**) $\boxed{\frac{4}{3}}$ (**D**) $\frac{5}{4}$ (**E**) None of these

47. Divide $1 - 7i$ by $6 - 2i$. Put your answer in $a + bi$ form.

(**A**) $\frac{2}{5} + \frac{4}{5}i$ (**B**) $-\frac{1}{4} - \frac{5}{4}i$ (**C**) $-\frac{1}{5} - \frac{11}{10}i$ (**D**) $\boxed{\frac{1}{2} - i}$ (**E**) None of these

48. Find the sum of all solutions to the equation

$$(x^3 + x^2 + 5x - 11)^2 - (4x^2 - 4x + 16)^2 = 0.$$

(**A**) $\boxed{-2}$ (**B**) 0 (**C**) -8 (**D**) 5 (**E**) None of these

49. Two committees consisting of 3 and 5 people, respectively, are to be formed from a group of 8 people. Assuming no person is on both committees, in how many ways can this be done?

(**A**) $\boxed{\frac{8!}{3!5!}}$ (**B**) $8!$ (**C**) $\frac{8!}{3!5!} \cdot \frac{8!}{5!3!}$ (**D**) $3!5!$ (**E**) None of these

50. Which of the following expressions is equivalent to

$$\log_2(x^3) - \log_2(xy^2) + 3\log_2(y^{1/2})?$$

(**A**) $\log_2(x^3 + y^{3/2} - xy^2)$ (**B**) $\log_2\left(\frac{x^2}{y^{7/2}}\right)$ (**C**) $\log_2(x^2y^{3/2})$

(**D**) $\boxed{\log_2\left(\frac{x^2}{y^{1/2}}\right)}$ (**E**) $\log_2(x^3 + 3y^{1/2} - xy^2)$