Alabama Statewide Math Contest - Round 4 Division 2

University of Alabama at Birmingham

April 6, 2019

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$,and $3 - 2\sqrt{3}$.

Round 4

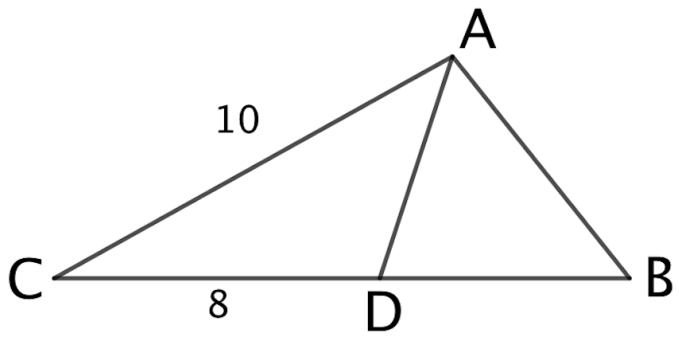
Geometry

Geometry Question # 1

Geometry Question # 1

RESET		

In the figure below, $AC = 10$, $CD = 8$, and $\angle CAD \cong \angle ABC$. Find the length of $\overline{BD}$.



Geometry Question # 1

Answer:

Geometry Question # 1

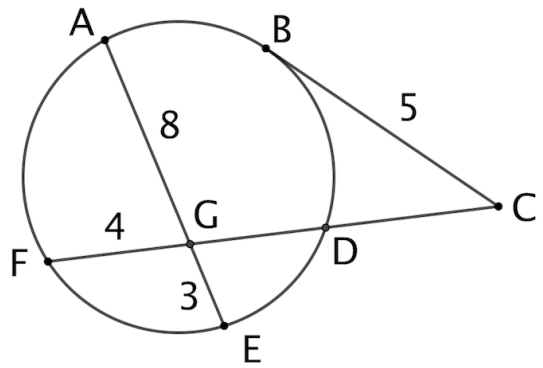
Answer: 4.5

Geometry Question # 2

Geometry Question # 2

RESET		

In the figure below, points A, B, D, E, and F are on the circle, $\overleftrightarrow{BC}$ is tangent to the circle at B with $BC = 5$, $\overline{CF}$ is a secant line, and $\overline{AE}$ and $\overline{DF}$ are chords intersecting at G with $AG = 8$, $FG = 4$ and $GE = 3$. Find CD.



Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $5\sqrt{2} - 5$

Round 4

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET		

Let $f^{-1}(x) = \dfrac{x^3 - 3}{5x^2 + 1}$. Find the y-intercept of the function $f(x)$.

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: $\sqrt[3]{3}$

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

How many zeros are at the end of 43! ?

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 9

Round 4

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET

The surface area of a sphere is 16π. What is the height of the shortest cylinder in which the sphere could be contained?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 4

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Find the value of $\log_{\sqrt{2}} 32$.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: 10

Round 4

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET		

If $3 - \sqrt{5}$ is one root of the quadratic function $f(x) = x^2 + bx + c$, where b and c are integers, find $b + c$.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: -2

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET		

Find the value of $\dfrac{71^2 - 29^2}{55^2 - 45^2}$.

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: 4.2

Round 4

Team

Team Question # 9

Team Question # 9

How many distinct arrangements are there of the letters STATEMATH? You should provide your answer as an integer.

Team Question # 9

Answer:

Team Question # 9

Answer: 30,240

Team Question # 10

Team Question # 10

RESET		

If $x - \frac{1}{x} = 2\sqrt{3}$, find the value of $\left(x^2 - \frac{1}{x^2}\right)^2$.

Team Question # 10

Answer:

Team Question # 10

Answer: 192

End of Round 4