Alabama Statewide Math Contest - Round 3 Division 2

University of Alabama at Birmingham

April 6, 2019

Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive 2 Bonus Points.

Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers π and e must be left as such.
4. Complex numbers must be put into $a + bi$ form.

Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8. $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
9. The time limit for **all** problems is 2 minutes.

Sample Problem # 1

Sample Problem

Solve for x in the equation

$$x^2 - 6x - 3 = 0$$

Sample Problem

Answer:

Sample Problem

Answer: $3 + 2\sqrt{3}$,and $3 - 2\sqrt{3}$.

Round 3

Geometry

Geometry Question # 1

Geometry Question # 1

RESET		

In triangle ABC, $BC = \sqrt{2}$, $CA = 6$ and the measure of $\angle ACB$ is 135°. What is the length of side $\overline{AB}$?

Geometry Question # 1

Answer:

Geometry Question # 1

Answer: $5\sqrt{2}$

Geometry Question # 2

Geometry Question # 2

RESET

Right triangle ABC has right angle at vertex C with $AC = 5$ and $AB = \sqrt{29}$. If the area of similar right triangle DEF is 45, find the perimeter of triangle DEF.

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: $21 + 3\sqrt{29}$

Round 3

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

Evaluate the expression $\sqrt[4]{81x^4y^3}$ if $x = -5$ and $y = 16$.

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: 120

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Find the largest solution to the equation $3 = 5x^{-1} + 12x^{-2}$.

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 3

Round 3

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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A polygon has twice as many diagonals as it has sides. How many sides does it have?

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 7

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET		

Solve for x:

$$\frac{1}{1-x}+\frac{1}{\sqrt{x}+1}+\frac{1}{\sqrt{x}-1}=0$$

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{1}{4}$

Round 3

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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An arithmetic sequence with first term of 3 and a last term of 59 has a sum of 465. How many terms are there in the sequence?

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: 15

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

A bucket contains one \$100 bill, two \$20 bills, and four \$1 bills. You select two bills at random, without replacement. What is the probability you got \$120 total?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{2}{21}$

Round 3

Team

Team Question # 9

Team Question # 9

RESET

The points $(2, 3)$ and $(4, 1)$ both lie on a circle with an arc of 60° between them. What is the radius of the circle?

Team Question # 9

Answer:

Team Question # 9

Answer: $2\sqrt{2}$

Team Question # 10

Team Question # 10

RESET		

Find the sum of all values of x on $[0, 2\pi)$ which satisfy $\cos 2x + 3\cos x - 1 = 0$.

Team Question # 10

Answer:

Team Question # 10

Answer: 2π

End of Round 3