Alabama Statewide Math Contest - Round 3 Division Two

University of North Alabama

April 9, 2016

Round 3

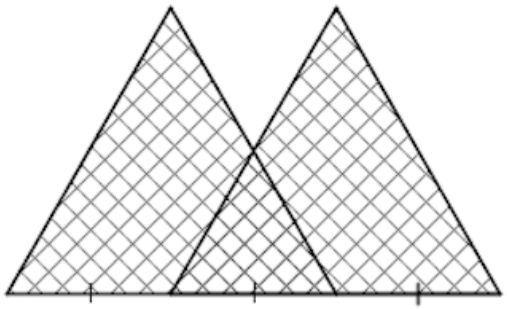
Geometry

Geometry Question # 1

Geometry Question # 1

RESET

Two equilateral triangles are positioned so that a vertex of one is the midpoint of a side of the other. If the side length is $2\sqrt{3}$, what is the area of the shaded region?



Geometry Question # 1

Answer:

Geometry Question # 1

Answer: $\frac{21\sqrt{3}}{4}$

Geometry Question # 2

Geometry Question # 2

RESET

A square is cut into 3 equal sized rectangles along 2 lines parallel to a side. If the perimeter of each is 24, then what is the area of the square?

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 81

Round 3

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET		

Simplify $(1/9 + 1/16)^{-1/2}$.

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: $\frac{12}{5}$

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Find the sum of the squares of the solutions to the equation

$$2x - \frac{3}{x} = 5$$

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 9.25

Round 3

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

RESET		

If $f(x) = 3x + 4$, find the point (a, b) where $f(x)$ intersects its inverse $f^{-1}(x)$.

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: $(-2, -2)$

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

RESET

Find all solutions x on $[0, 2\pi)$ such that $3\cos^2 x - 6 = \cos x - 2$.

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: π

Round 3

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

RESET

Find the value of $(2+i)(3-4i)(2-i)$, where $i=\sqrt{-1}$. Put your answer in standard form.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: $15 - 20i$

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET		

For what constant k will the line $(k+7)x - 3y = 1$ have slope k?

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $\frac{7}{2}$

Round 3

Team

Team Question # 9

Team Question # 9

RESET		

What is the distance from the origin to the line $3y - x = 30$?

Team Question # 9

Answer:

Team Question # 9

Answer: $3\sqrt{10}$

Team Question # 10

Team Question # 10

RESET		

Solve for x in the following equation

$$\log_3 x + \log_3 x^2 + \log_3 x^3 + \cdots + \log_3 x^{63} = 2016$$

Team Question # 10

Answer:

Team Question # 10

Answer: 3

End of Round 3