Alabama Statewide Math Contest - Round 2 Division Two

University of North Alabama

April 9, 2016

Round 2

Geometry

Geometry Question # 1

Geometry Question # 1

RESET

A box has dimensions x, $2x - 2$ and $3x$. If the box has a surface area of 150, find the volume of the box.

Geometry Question # 1

Answer:

Geometry Question # 1

Answer: 108

Geometry Question # 2

Geometry Question # 2

RESET

An isosceles right triangle is inscribed in a circle. If the area of the triangle is 36, what is the circumference of the circle?

Geometry Question # 2

Answer:

Geometry Question # 2

Answer: 12π

Round 2

Algebra II & Trig

Algebra II & Trig Question # 3

Algebra II & Trig Question # 3

RESET

Define an operation $\rtimes$ on pairs of numbers as $(a, b) \rtimes (c, d) = (ac, b + d)$. Find (a, b) so that

$$(a, b) \rtimes (-1, 3) = (2, 5)$$

Algebra II & Trig Question # 3

Answer:

Algebra II & Trig Question # 3

Answer: $(-2, 2)$

Algebra II & Trig Question # 4

Algebra II & Trig Question # 4

RESET

Find the sum of the squares of all solutions the equation $\sqrt[3]{x-1} = x - 1$.

Algebra II & Trig Question # 4

Answer:

Algebra II & Trig Question # 4

Answer: 5

Round 2

Comprehensive Part 1

Comprehensive Part 1
Question # 5

Comprehensive Part 1 Question # 5

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Find the largest solution of the equation $(2x - 1)(3x + 4) = 15x$.

Comprehensive Part 1 Question # 5

Answer:

Comprehensive Part 1 Question # 5

Answer: 2

Comprehensive Part 1
Question # 6

Comprehensive Part 1 Question # 6

You roll two ten-sided dice. What is the probability that the dice sum to less than or equal to 6?

Comprehensive Part 1 Question # 6

Answer:

Comprehensive Part 1 Question # 6

Answer: $\frac{3}{20}$

Round 2

Comprehensive Part 2

Comprehensive Part 2
Question # 7

Comprehensive Part 2 Question # 7

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Find the largest value in $[0, 2\pi)$ satisfying $\sin 2x = \frac{1}{2}$.

Comprehensive Part 2 Question # 7

Answer:

Comprehensive Part 2 Question # 7

Answer: $\dfrac{17\pi}{12}$

Comprehensive Part 2
Question # 8

Comprehensive Part 2 Question # 8

RESET

If $h(x) = 5x^2 - 3x$, find the largest solution to the equation $h(x + 2) = 0$.

Comprehensive Part 2 Question # 8

Answer:

Comprehensive Part 2 Question # 8

Answer: $-\frac{7}{5}$

Round 2

Team

Team Question # 9

Team Question # 9

RESET		

Put the following numbers in order from least to greatest:

$$2^{30} \quad 3^{40} \quad 5^{20} \quad 7^{10}$$

Team Question # 9

Answer:

Team Question # 9

Answer: $7^{10}, 2^{30}, 5^{20}, 3^{40}$

Team Question # 10

Team Question # 10

RESET		

Name three regular polygons that can tessellate the Euclidean plane.

Team Question # 10

Answer:

Team Question # 10

Answer: Triangle, Square, Hexagon

End of Round 2