The 43rd Annual

$\mathcal{ALABAMA}$

STATEWIDE MATHEMATICS CONTEST

ACTM

Written Round: February 24, 2024 at Regional Testing Sites

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GEOMETRY EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E. A calculator is NOT permitted.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.

- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.

- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B

- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.

- If A and B are points on a circle, then $\overarc{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\overarc{AB}$ is the measure of $\overarc{AB}$ in degrees.

- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.

- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.

- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.

- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Mining

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

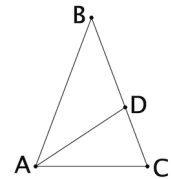1. A square $ABCD$ is 5 units on each side and is inscribed in a circle with center O. Find the area of the circle.

   (**A**) $\dfrac{24\pi}{4}$ (**B**) $\boxed{\dfrac{25\pi}{2}}$ (**C**) 25π (**D**) 50π (**E**) None of these

2. If a regular polygon has nine diagonals, how many sides does it have?

   (**A**) $\boxed{6}$ (**B**) 7   (**C**) 8   (**D**) 9   (**E**) None of these

3. In the following figure, $AB = BC$ and $AC = AD = BD$. Determine the measure of $\angle ADB$ in degrees.

   (**A**) $72°$ (**B**) $\boxed{108°}$ (**C**) $136°$ (**D**) $148°$ (**E**) None of these

4. The graph of $|2x| + |y| = 4$ encloses a region of the plane. What is the area of the region?

   (**A**) $\boxed{16}$ (**B**) 20   (**C**) 32   (**D**) 40   (**E**) None of these

5. Let $\triangle ABC$ be a triangle with sides $AB = 6$, $AC = 12$ and $BC = 6\sqrt{5}$. Find the length of the median corresponding to $\overline{BC}$.
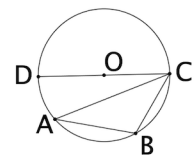
   (**A**) 3   (**B**) 6   (**C**) $\boxed{3\sqrt{5}}$ (**D**) $\dfrac{12\sqrt{5}}{5}$ (**E**) None of these

6. The supplement of an angle is $30°$ more than four times its complement. Find the measure of the angle.

   (**A**) $58°$ (**B**) $62°$ (**C**) $\boxed{70°}$ (**D**) $74°$ (**E**) None of these

7. In the diagram, O is the center of the circle, $\overline{DC}$ is a diameter, and $m\angle BCD = 55°$. Find $m\angle BAC$.

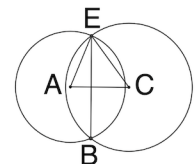   (**A**) $\boxed{35°}$ (**B**) $45°$ (**C**) $55°$ (**D**) $65°$ (**E**) None of these

8. Rectangle R_1 has one side of 2 inches and area 12 square inches. Rectangle R_2 with diagonal of 15 inches is similar to rectangle R_1. What is the area of rectangle R_2 in square inches?

   (**A**) $\boxed{\dfrac{135}{2}}$ (**B**) $\dfrac{225}{8}$ (**C**) 90   (**D**) 288   (**E**) None of these

9. In the figure shown, the circles centered at A and C intersect at points E and B. If $\overline{EB}$ is perpendicular to $\overline{AC}$, $AE = 26$, $AC = 28$, and $CE = 30$, find BE.

   (**A**) 10   (**B**) 20   (**C**) 24   (**D**) $\boxed{48}$ (**E**) None of these

10. Let $\triangle ABC$ be a triangle with angle $\angle ACB$ a right angle, $m\angle ABC = 30°$, and D on $\overline{AB}$ such that $\overline{CD}$ is an altitude from C to base $\overline{AB}$. If $CD = 9$, find AB.

   (**A**) 12   (**B**) 18   (**C**) $\boxed{12\sqrt{3}}$ (**D**) $18\sqrt{3}$ (**E**) None of these

11. A right triangle has a perimeter of 32 units and an area of 20 square units. Find the length of the hypotenuse.
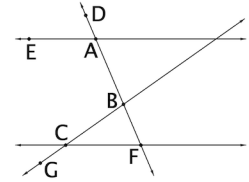
(**A**) $\dfrac{26}{3}$ (**B**) $\boxed{\dfrac{59}{4}}$ (**C**) $\dfrac{69}{4}$ (**D**) $\dfrac{70}{3}$ (**E**) None of these

12. What is the measure of each exterior angle of a regular octagon?

(**A**) $35°$ (**B**) $\boxed{45°}$ (**C**) $50°$ (**D**) $60°$ (**E**) None of these

13. In the diagram shown, G lies on $\overleftrightarrow{BC}$, B and D lie on $\overleftrightarrow{AF}$, $\overleftrightarrow{EA} \parallel \overleftrightarrow{CF}$, $m\angle EAD = 32°$, and $m\angle ABC = 104°$. Find $m\angle FCG$.

(**A**) $72°$ (**B**) $\boxed{108°}$ (**C**) $136°$ (**D**) $148°$ (**E**) None of these

14. A equilateral triangle is inscribed in a circle with radius of 5. What is the perimeter of the equilateral triangle?
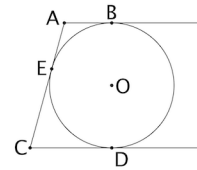
(**A**) 15 (**B**) 30 (**C**) $\dfrac{15\sqrt{3}}{2}$ (**D**) $\boxed{15\sqrt{3}}$ (**E**) None of these

15. If the graphs of $4y + 2x + 6 = 0$ and $6y + ax + 4 = 0$ intersect at a right angle, what is the value of a?

(**A**) $\boxed{-12}$ (**B**) -3 (**C**) 3 (**D**) 12 (**E**) None of these

16. In the diagram shown, $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{CD}$ are tangent to the circle at points B, E, and D, respectively. If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $AE = 4$, and $EC = 9$, what is the radius of the circle?

(**A**) $\dfrac{5}{2}$ (**B**) $2\sqrt{5}$ (**C**) $\sqrt{65}$ (**D**) $\boxed{6}$ (**E**) None of these

17. Nine times the reciprocal of the circumference of a circle is equal to one-third the diameter of the circle. What is the radius of the circle?

(**A**) $\dfrac{9\sqrt{\pi}}{\pi}$ (**B**) $\dfrac{3\sqrt{3\pi}}{\pi}$ (**C**) $\dfrac{9\sqrt{\pi}}{2\pi}$ (**D**) $\boxed{\dfrac{3\sqrt{3\pi}}{2\pi}}$ (**E**) None of these
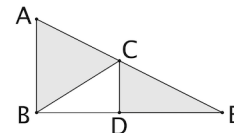
18. A flagpole has cracked 18 feet from the ground and has fallen as if hinged. The top of the flagpole hits the ground 24 feet from the base. How tall was the flagpole before it fell?

(**A**) 42 feet   (**B**) 46 feet   (**C**) $\boxed{48 \text{ feet}}$ (**D**) 50 feet   (**E**) None of these

19. A rectangular concrete patio is to be built 18 feet long, 9 feet wide and 8 inches thick. How many cubic yards of concrete are needed?

(**A**) $\boxed{4}$ (**B**) $\dfrac{9}{2}$ (**C**) $\dfrac{81}{2}$ (**D**) 48 (**E**) None of these

20. In the given right triangle $\triangle ABE$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$, $AB = 13$, $BD = 12$, and $DE = 14$. What is the area of the shaded region?

(**A**) 98   (**B**) $\boxed{127}$ (**C**) 143   (**D**) $\dfrac{338}{3}$ (**E**) None of these

21. If the diagonals of a rhombus are length 16 and 24, find the perimeter of the rhombus.

(**A**) $16\sqrt{11}$ (**B**) $\boxed{16\sqrt{13}}$ (**C**) $16\sqrt{15}$ (**D**) $16\sqrt{17}$ (**E**) None of these
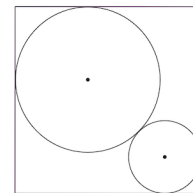
22. Find the size of the smaller angle of a non-right isosceles triangle in which one angle is half the size of another.

(**A**) $\boxed{36°}$ (**B**) $40°$ (**C**) $72°$ (**D**) $80°$ (**E**) None of these

23. A tangent and a secant are drawn to a circle from an external point A. The point of tangency is B with $AB = 14$, and the secant intersects the near side of the circle at C and the far side at D. If CD and AC are in the radio of $3 : 1$, what is the length of $\overline{AD}$?

(**A**) $\boxed{28}$ (**B**) $\dfrac{28\sqrt{3}}{3}$ (**C**) 56 (**D**) $\dfrac{56\sqrt{3}}{3}$ (**E**) None of these

24. The circles in the figure each have radius 1 and 2, respectively, and their centers lie along the diagonal of the square. Each circle is tangent to two sides of the square as shown and they are tangent to one another. Determine the area of the square.
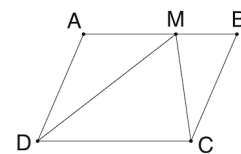
(**A**) $3 + 3\sqrt{2}$ (**B**) $6 + \sqrt{2}$ (**C**) $19 + 6\sqrt{2}$

(**D**) $12 + 3\sqrt{2}$ (**E**) $\boxed{\text{None of these: } \frac{27}{2} + 9\sqrt{2}}$

25. Two vertical poles, 16 feet and 24 feet high, are erected 13 feet apart on level ground. Straight wires are attached from the top of each pole to the bottom of the other pole. Find the vertical distance from the ground to the intersection of the wires.

(**A**) 5.2 feet   (**B**) 7.8 feet   (**C**) $\boxed{9.6 \text{ feet}}$ (**D**) 10.2 feet   (**E**) None of these

26. Let $ABCD$ be a parallelogram and M be a point on the segment $\overline{AB}$ such that the area of $\triangle MBC = 8$ and the area of $\triangle MDC = 20$. Find the area of parallelogram $ABCD$.

(**A**) 32   (**B**) 36   (**C**) $\boxed{40}$ (**D**) 44   (**E**) None of these

27. The altitude of a pyramid with a square base is 16. A cross section taken 10 inches below the vertex and parallel to the base has an area of 56.25 square inches. What is the area of the base of the pyramid in square inches?
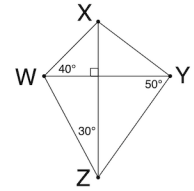
(**A**) 90   (**B**) 100   (**C**) 128   (**D**) $\boxed{144}$ (**E**) None of these

28. Let A and B be the points of intersection of the circles $x^2 + y^2 - 6x + 4y = 3$ and $x^2 + y^2 + 4x - 4y = 17$. Determine the slope of the segment $\overline{AB}$.

(**A**) $\boxed{\dfrac{5}{4}}$ (**B**) $\dfrac{4}{5}$ (**C**) $-\dfrac{5}{4}$ (**D**) $-\dfrac{4}{5}$ (**E**) None of these

29. In the figure, $\overline{XZ} \perp \overline{WY}$, $m\angle WZX = 30°$, $m\angle XWY = 40°$, $m\angle WYZ = 50°$. What is the measure of $\angle ZXY$?

(**A**) 30°   (**B**) 40°   (**C**) 50°   (**D**) 60°   (**E**) None of these

30. A circle passes through points $(1, 2)$, $(3, -4)$, and $(5, -6)$. Find the coordinates of the center of the circle.
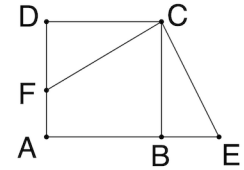
(**A**) $(10, 1)$ (**B**) $(11, 2)$ (**C**) $(12, 0)$ (**D**) $(13, 3)$ (**E**) None of these

31. Two circles of radii 3 and 9, centered at points A and B, respectively, are externally tangent. Let $\overline{CD}$ be the common external tangent with C on the circle centered at A and D on the circle centered at B. Find the length of $\overline{BC}$.

(**A**) $3\sqrt{13}$ (**B**) $3\sqrt{17}$ (**C**) $3\sqrt{19}$ (**D**) $3\sqrt{21}$ (**E**) None of these

32. In square $ABCD$, F is a point on side $\overline{AD}$ and E is a point on the extension of side $\overline{AB}$ such that $\overline{CF}$ and $\overline{CE}$ are perpendicular. If each side of the square has a length of 20 and the area of triangle $\triangle CEF$ is 288, what is the area of triangle $\triangle AFE$?

(**A**) 56   (**B**) 72   (**C**) 112   (**D**) 144   (**E**) None of these

33. Five test scores have a mean of 91, a median of 92, and a mode of 95. Find the sum of the two lowest test scores.

(**A**) 177   (**B**) 173   (**C**) 171   (**D**) 168   (**E**) None of these

34. The point $A = (2, 3)$ is reflected about the x-axis to a point B. The point B is then reflected about the line $y = x$ to a point C. What is the distance from point A to point C?

(**A**) $\sqrt{26}$ (**B**) $\sqrt{29}$ (**C**) $\sqrt{34}$ (**D**) $2\sqrt{5}$ (**E**) None of these

35. Find the area of a square in which the diagonal is six feet longer than the side.

(**A**) 36   (**B**) 108   (**C**) $36 + 72\sqrt{2}$ (**D**) $108 + 72\sqrt{2}$ (**E**) None of these
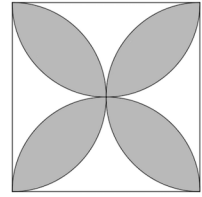
36. The perimeters of a regular hexagon and a square are equal. If α is the area of the hexagon and β is the area of the square, what is α/β?

(**A**) $\sqrt{3}$ (**B**) $\dfrac{\sqrt{3}}{2}$ (**C**) $\dfrac{3\sqrt{3}}{2}$ (**D**) $\dfrac{2\sqrt{3}}{3}$ (**E**) None of these

37. If $\sin\theta = \dfrac{3}{4}$ and $\cos\theta \geq 0$, what is $\cos\theta$?

(**A**) $\dfrac{4}{5}$ (**B**) $\dfrac{\sqrt{5}}{4}$ (**C**) $\dfrac{\sqrt{7}}{4}$ (**D**) $\dfrac{\sqrt{7}}{5}$ (**E**) None of these

38. In the figure shown, each petal is formed by the intersection of semicircles which connect to the corners of a square with side length 10. Find the area of the shaded region.
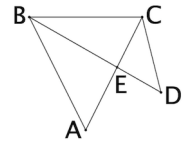
(**A**) $100 - \dfrac{25\pi}{2}$ (**B**) $100 - 25\pi$ (**C**) $25\pi - 50$

(**D**) $\boxed{50\pi - 100}$ (**E**) None of these

39. A right triangle has legs measuring $x - 5$ and $x + 4$ with a hypotenuse of $x + 7$. Find the perimeter of this triangle.

(**A**) $6 + 6\sqrt{2}$ (**B**) $6 + 12\sqrt{2}$ (**C**) $6 + 18\sqrt{14}$ (**D**) $30 + 6\sqrt{14}$ (**E**) $\boxed{30 + 18\sqrt{2}}$

40. In the figure shown, $\overline{AB} \cong \overline{AC}$, $\overline{EC} \cong \overline{ED}$, and $\overline{BC} \cong \overline{DC}$. If $m\angle BEC = 86°$, find the measure of $\angle CAB$.

(**A**) $35°$ (**B**) $43°$ (**C**) $\boxed{78°}$ (**D**) $94°$ (**E**) None of these

41. A right prism with equilateral triangle bases has a volume of $120\sqrt{3}$. If the distance between bases is 10, what is the surface area of the prism?

(**A**) $\boxed{144\sqrt{3}}$ (**B**) $204\sqrt{3}$ (**C**) $24\sqrt{3} + 120$ (**D**) $24\sqrt{3} + 180$ (**E**) None of these

42. Let A, B, and C be on a circle such that the center O lies on $\overline{AC}$. If $AB = 36$, and $BC = 15$, what is the length of arc $\overparen{AC}$?

(**A**) $\boxed{\dfrac{39}{2}\pi}$ (**B**) $\dfrac{41}{2}\pi$ (**C**) 78π (**D**) 82π (**E**) None of these

43. Triangle $\triangle ABC$ has $m\angle BAC = 2x - 15$, $m\angle ABC = 3x + 30$ and the measure of the exterior angle at vertex C equal to $x + 69$. What is the $m\angle ACB$? Assume all measurements are in degrees.

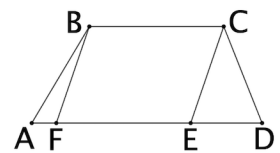(**A**) $70.5°$ (**B**) $85°$ (**C**) $90°$ (**D**) $\boxed{97.5°}$ (**E**) None of these

44. Let $ABCD$ be a convex quadrilateral with diagonals of length 18 and 28. Let E, F, G, H be the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Find the perimeter of quadrilateral $EFGH$.

(**A**) $4\sqrt{115}$ (**B**) $4\sqrt{227}$ (**C**) $\dfrac{184}{5}$ (**D**) $\dfrac{92}{3}$ (**E**) $\boxed{\text{None of these: 46}}$

45. Let A, C, B, D be points on a circle such that secant $\overline{AB}$ intersects secant $\overline{CD}$ at a point E where $CE = 2AE$, $AB = 12$ and $CD = 10$. Find AE.

(**A**) $\dfrac{5}{2}$ (**B**) $\boxed{\dfrac{8}{3}}$ (**C**) $\dfrac{16}{5}$ (**D**) $\dfrac{60}{17}$ (**E**) None of these

46. A parallelogram $\square BCEF$ is inscribed into trapezoid $ABCD$ as shown in the figure with $EC = CD$. If $AF = 4$, $BC = 10$, $ED = 12$, and $m\angle BAF = 30°$, find the area of $\square ABCD$.

(**A**) $\dfrac{104\sqrt{3}}{2}$ (**B**) $\dfrac{260\sqrt{3}}{3}$ (**C**) $24\sqrt{3}$ (**D**) $\boxed{60\sqrt{3}}$ (**E**) None of these

47. Which of the following shapes have at least one pair of congruent sides?

    I. Parallelogram   II. Isosceles Triangle   III. Trapezoid   IV. Hexagon

(**A**) $\boxed{\text{I and II}}$ (**B**) I and III   (**C**) III and IV   (**D**) I, II, and IV   (**E**) None of these

48. What is the area of the triangle whose vertices are given by the points $(1, 2)$, $(4, 6)$, and $(5, 5)$?

(**A**) $\boxed{\dfrac{7}{2}}$ (**B**) $\dfrac{5\sqrt{2}}{2}$ (**C**) $8\sqrt{6}$ (**D**) $\dfrac{\sqrt{46}}{2}$ (**E**) None of these

49. What is the image of the point $(2, 2)$ after a $45°$ counterclockwise rotation about the origin?

(**A**) $(0, 2)$ (**B**) $(0, 4)$ (**C**) $\boxed{(0, 2\sqrt{2})}$ (**D**) $(0, 4\sqrt{2})$ (**E**) None of these

50. How many different triangles with integer side lengths can be created with two sides of length 14 and 19?

(**A**) 26   (**B**) $\boxed{27}$ (**C**) 31   (**D**) 32   (**E**) None of these